

# Optimization of Cellular Networks

## Capacity and Densification

### An Analytical approach

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#### ABSTRACT

We propose an analytical approach of cellular networks, which allows to optimize transmitting powers of base stations, and to maximize network capacity. In our framework, mobiles are considered as a continuum. Networks are often densified *i.e.* new base stations are added because of increasing traffic. Our approach enables to analyse the extra capacity offered by each new base station, according to its location. It can be performed without any simulation whatever the model used for the propagation. The continuous approach proposed in this paper can be applied to any frequency reuse 1 networks, such as OFDMA or CDMA ones. As an example, we calculate the maximum number of active mobiles per cell in a homogeneous network, considering different kinds of environments, and we show how to optimize the densification of a network.

#### General Terms

Performance, Theory

#### Keywords

analytical model, CDMA, OFDMA, densification, optimization.

## 1. INTRODUCTION

In order to enable a frequency reuse 1 cellular network to support more traffic, a telecom provider can choose to *densify* the network, i.e to add new base stations (BS). In order to analyse the advantages and drawbacks of this solution, the provider generates simulations with relevant tools. These simulations do not give instantaneous results, may last a long time, and moreover, must be repeated many times by varying conditions. We develop thereafter an analytical approach. This approach can be applied to any frequency reuse 1 network, such as CDMA and OFDMA. It enables to analyse and compare *instantly* different solutions in the aim to adapt the network, or a given zone of the network, to an increasing traffic demand.

For clarity of presentation, this paper is focused on CDMA networks. However, the analysis we develop can be used for other technologies such as OFDMA (see remark at the end of the section 2). Classical CDMA networks models [1-2] do not give explicit and simple analytical expressions due to the complexity of the analysis: for the downlink, the interferences received by a mobile are due to all the base stations of the network. They

depend on their transmission powers, positions and numbers. We develop an analytical network's approach which does not need any simulation to obtain explicit expressions of some important characteristics of the network such as the possibility for a mobile to be admitted in the network. Our model is based on the assumption that mobiles are uniformly distributed in each cell, which enables the transformation of the discrete sum of constraints on a mobile to its mean value, expressed as an integral. The problem of CDMA capacity constraints has already been considered by several authors. Nettleton and Alavi [8] considered the power allocation problem in the cellular spread spectrum context. Considering each user has a required bit rate, Gilhousen et al [7] assumed a capacity of the network only interference-limited. In our paper, we propose an analytical method to calculate the maximum capacity of a cell or a network, using the minimum transmitting power. As a result of this approach, the maximum number of active mobiles supported in a network will only depend on the characteristics of the network: propagation conditions and positions of the base stations.

The paper is organized as follows. Expressing the SIR target constraints for a mobile to be connected to a BS of a CDMA network, we establish the matrixial inequality of any transmitted power to any active mobile of the network. Establishing the conditions under which the inequality has a solution, we propose afterwards a method to optimize these powers. We obtain the analytical expression of the capacity of a cell, and analyze the densification consequences.

## 2. Analysis of cellular networks

We use the model similar to [4]. Let us consider a network of  $N_{BS}$  base stations, each  $BS_j$  defining a cell  $j$ . As an example, we develop our analysis considering a CDMA network. We assume that each mobile is active. We express that the *Signal to Interference Ratio (SIR)* received by a mobile has to be at least equal to a minimum threshold target value  $\gamma$  [4] [5]. The interferences are due to an intra-cell interference  $I_{own}$  which represents the interferences due to the common channels and the traffic channels of the other mobiles located in the cell  $b$ , an inter-cell interference  $I_{other}$  which represents the interferences due to the other base stations of the network, and the level of the thermal noise  $N_{th}$  at the receiver of the mobile. Using the equation of the

transmission traffic channel power [4] for the downlink, the following condition has to be satisfied:

$$\frac{(P_{b,m}/g_{b,m})}{\alpha(P_b - P_{b,m})/g_{b,m} + \sum_{j=1, j \neq b}^{N_{BS}} P_j/g_{j,m} + N_{th}} \geq \gamma \quad (\text{II.1})$$

where

$P_b$  is the total transmission power of the base station  $b$ , including the common channels; each one of these common channels is assumed to be orthogonal to all other channels.

$\gamma$  represents the level of ratio signal on interference target for the service used by mobile. For simplicity of presentation, we assume a unique value of  $\gamma$ .

$P_{j,m}$  is the transmitting power towards the mobile  $m$  belonging to the cell  $j$ ;

$P_j$  is the total transmitting power of the BS  $j$ ;

$g_{n,m}$  is the total pathloss between the mobile  $m$  and the BS  $n$ , i.e. the relationship between the power transmitted by the traffic channel of  $j$  towards  $m$  and the power received by the receiver of the mobile  $m$ . This term thus contains a pathloss depending on the distance, the shadowing, losses and profits of antenna of mobile  $m$  and the base station  $j$  and possible additional losses;

$\alpha \in [0,1]$  is the orthogonality factor, which can be determined by model [9]. It characterizes the loss of orthogonality between signals transmitted simultaneously.  $\alpha = 1$  if there is no orthogonality between the channels resulting from the same base station, and  $\alpha = 0$  if the orthogonality is perfect. In the following, we assume that the orthogonality factor is the same for the whole network.

Introducing the notation

$$\beta = \frac{\gamma}{1 + \alpha\gamma}, \quad (\text{II.2})$$

(II.1) can be expressed as follows.

$$P_{b,m} \geq \beta(\alpha P_b + g_{b,m} \sum_{j \neq b}^{N_{BS}} \frac{1}{g_{j,m}} P_j + g_{b,m} N_{th}) \quad (\text{II.3})$$

This expression shows that the power needed by a mobile belonging to the BS  $b$  depends on all the BS transmitting powers of the network. It however *does not depend* on each dedicated transmitting power of any given BS  $j$  of the network to a given mobile  $m$  belonging to  $j$ : it only depends on the *total transmitting power* of the BS  $j$ . The total power of the serving base station  $b$  can be calculated from the equation (II.3) as:

$$P_b = P_{b,CCH} + \sum_{m=1}^{N_b^{MS}} P_{b,m} \quad (\text{II.4})$$

where

$N_j^{MS}$  represents the mobiles number of the cell  $j$ , and

$P_{b,CCH}$  represents the power dedicated to the common channel.

We can generalize the expression (II.3) for each mobile  $m$  belonging to any cell  $n$  of the network:

$$P_{n,m} \geq \beta(\sum_{k=1}^{N_j^{MS}} \sum_{j=1}^{N_{BS}} \frac{g_{n,m}}{g_{j,m}} \alpha_{j,n} P_{j,k} + \sum_{j=1}^{N_{BS}} \frac{g_{n,m}}{g_{j,m}} \alpha_{j,n} P_{j,CCH} + g_{n,m} N_{th}) \quad (\text{II.5})$$

where

$$\alpha_{j,n} = 1 \quad \text{if} \quad j \neq n$$

$$\alpha_{j,n} = \alpha \quad \text{if} \quad j = n$$

Generalizing (II.5) for each mobile and for each BS of the network leads to the following system:

$$P \geq AP + B \quad (\text{II.6})$$

$P$  represents the set of transmission powers towards each mobile of the network. It can be written as

$$P = (P_{11}, P_{12}, \dots, P_{1N_1}, P_{21}, P_{22}, \dots, P_{2N_2}, \dots, P_{j1}, P_{j2}, \dots, P_{jN_j}, \dots, P_{N_{BS}1}, P_{N_{BS}2}, \dots, P_{N_{BS}N_{BS}})$$

For example  $P_{11}$  represents the power transmitted to the mobile number  $1$  by the base station number  $1$ .

Let  $A = (a_{mk})$  and  $B = (b_{nm})$  then:

$$a_{mk} = \alpha\beta \quad \text{if the mobiles } m \text{ and } k \text{ are served by the same BS}$$

$$a_{mk} = \beta \frac{g_{n,m}}{g_{j,m}} \quad \text{if the mobiles } m \text{ is served by a BS } n \text{ and the mobile } k \text{ is served by a BS } j \text{ with } j \neq n.$$

$$\text{And } b_{nm} = \beta g_{n,m} (\sum_{j=1}^{N_{BS}} \frac{1}{g_{j,m}} \alpha_{j,n} P_{j,CCH} + N_{th})$$

The elements of the matrix  $A$  depend on the pathloss ratios received by the mobiles and their quality of service characterized by the parameter  $\beta_m$ . For a real network, they can be determined using different measures such as the power transmissions and power receptions. Relation (II.6) expresses a relation between the power transmission needed by each mobile in the network and the total powers of all the BS of the network. The term  $a_{mk} = \alpha\beta$  characterizes the intracell interferences  $I_{own}$ , and the term

$$a_{mk} = \beta \frac{g_{n,m}}{g_{j,m}} \quad \text{characterizes the inter cell interferences } I_{\text{other}}.$$

In order to optimize all transmitting powers, we have to determine the minimum power needed by each mobile of each cell, i.e. to solve (II.6). Each mobile entering a cell of the network modifies the system characterized by (II.6), and requires a new resolution. We develop hereafter a method to determine the minimum BS transmitting powers.

**Remark:**

Though our analysis is focused on CDMA networks, the model we develop is still valid for cellular technologies without internal interference, providing that  $I_{\text{own}} = 0$ . It can be applied, in particular, to frequency reuse 1 networks based on other technologies, such as OFDMA.

### 3. ANALYTICAL MODEL

We redefine the indexes as follows:

Let

$$P = \begin{pmatrix} P_1 \\ \vdots \\ P_N \end{pmatrix} \quad (\text{III.1})$$

the power vector.

Each element of this vector represents a transmitting power towards a mobile of the network: the network manages  $N$  mobiles.

$P$  must verify (II.6) where

$$A = \begin{pmatrix} a_{11} \cdots a_{1N} \\ \cdots \cdots \cdots \\ a_{N1} \cdots a_{NN} \end{pmatrix}$$

and

$$B = \begin{pmatrix} b_1 \\ \vdots \\ b_N \end{pmatrix}.$$

All  $a_{mk}$  and  $b_m$  are positive, and all  $P_m$  must be positive.

The conditions under which there exist finite solutions are given in [6].

#### 3.1 Optimal Solutions

An optimum solution may be expressed as a solution that minimizes a function  $h$  such as  $h(P_1, \dots, P_n)$ . In practice,  $h$  is a linear function, and generally  $h(P_1, \dots, P_n) = P_1 + \dots + P_n$ .

**Proposition 1:**

If  $h$  is a quasi-concave function, and if there is a solution, then this solution is on the edge of the domain.

**Proof:**

The domain defined by  $P \geq AP + B$  and  $P_i \geq 0$  is a convex set (since it is an intersection of convexes). Let assume that the minimum of  $g$  is on  $X$  interior of the domain. Then, in the neighborhood of  $X$ , the *iso-h* sets are closed surfaces around  $X$ . Any line passing through  $X$  will intersect such a surface in two points, say  $Y$  and  $Z$ .  $h(Y) = h(Z) \geq h(X)$ , which contradicts the hypothesis of  $h$  being a quasi-concave function.

**Proposition 2:**

If the spectral radius of  $A$  is strictly smaller than 1, then the system has a solution.

**Proof:**

$$(I - A)(I + A + A^2 + \cdots + A^k)B = (I - A^{k+1})B$$

$$(I - A)(I + A + A^2 + \cdots + A^k)B = B - A^{k+1}B$$

Since 1 is not an eigenvalue of  $A$ ,  $I - A$  is invertible. Therefore,

$$(I + A + A^2 + \cdots + A^k)B = (I - A)^{-1}(B - A^{k+1}B)$$

$$(I + A + A^2 + \cdots + A^k)B = (I - A)^{-1}B - (I - A)^{-1}A^{k+1}B$$

Since the spectral radius of  $A$  is strictly smaller than 1,

$$\lim_{k \rightarrow +\infty} (A^{k+1}) = 0, \text{ and } \lim_{k \rightarrow +\infty} ((I - A)^{-1}A^{k+1}B) = 0$$

Therefore,  $(I + A + A^2 + \cdots + A^k)B$  is convergent and tends to  $(I - A)^{-1}B$

Since all components of  $(I + A + A^2 + \cdots + A^k)B$  are positive, all components of the limit are positive.

Besides, one can easily prove, by recurrence, that, for all  $k$ :

$$\begin{cases} P \geq AP + B \\ P \geq 0 \end{cases} \Rightarrow P \geq (I + A + A^2 + \cdots + A^k)B$$

By making  $k$  tend to infinity:

$$\begin{cases} P \geq AP + B \\ P \geq 0 \end{cases} \Rightarrow P \geq (I - A)^{-1}B$$

This latter relation means that any solution of (II.6) is greater than  $(I - A)^{-1}B$ , which itself is obviously a solution.

Therefore, the solution that minimizes  $h$  is

$$(I - A)^{-1}B = (I + A + A^2 + \cdots + A^k + \cdots)B \quad (\text{III.2})$$

The complexity of calculation of the right member of this equality, truncated at rank  $k$ , is  $kn^2$ .

**Remark:**

For a given vector  $B$ , the proposition 2 gives a *sufficient condition* for the system to have a solution; however this condition is necessary and sufficient that, for all  $B$ , the system has a solution.

**Proposition 3:**

If, for all  $m$ ,  $\sum_k a_{mk} \leq \lambda$ , then the spectral radius of  $A$  is less than  $\lambda$ .

**Proof:**

Let  $u$  be an eigenvector of  $A$ ,  $\omega$  its eigenvalue, and  $j = \text{argmax}_m(|u_m|)$ . The  $j^{\text{th}}$  element of  $Au$  can be written:

$$\omega u_j = \sum_k a_{jk} u_k$$

$$|\omega u_j| = \left| \sum_k a_{jk} u_k \right| \leq \sum_k a_{jk} |u_k| \leq |u_j| \sum_k a_{jk} \leq \lambda |u_j|$$

Therefore,  $|\omega| \leq \lambda$ . Since all the eigenvalues modules are less than  $\lambda$ , the spectral radius of  $A$  is less than  $\lambda$ .

**Proposition 4:**

If, for all  $k$ ,  $\sum_m a_{mk} \leq \lambda$ , then, the spectral radius of  $A$  is less than  $\lambda$ .

**Proof:**

By applying the proposition 3 to  ${}^t A$ , we see that the spectral radius of  ${}^t A$  is less than  $\lambda$ . Therefore, the spectral radius of  $A$  is less than  $\lambda$ .

### 3.2 Continuous approach

Let  $A = (a_{mk})_{1 \leq m, k \leq N}$ , then:

$a_{mk} = \alpha\beta$  if mobiles  $m$  and  $k$  are served by the same BS;

$a_{mk} = \beta \frac{g_{n,m}}{g_{j,m}}$  if mobile  $m$  is served by BS  $n$  and mobile  $k$  is

served by BS  $j$  with  $j \neq n$ .

Therefore, for all BS  $j$ , for all  $k$  served by  $j$

$$\sum_m a_{mk} = \sum_{i \in j} \alpha\beta + \sum_{n \neq j} \sum_{m \in n} \beta \frac{g_{n,m}}{g_{j,m}} \quad (\text{III.3})$$

Let's call  $M_m$  the position of mobile  $m$ , and  $H_n$  the cell served by base station  $n$ .

If, for all  $m, j$ ,  $g_{j,m} = g_j(M_m)$ , and under the assumption that  $N_n$  mobiles are uniformly distributed in  $H_n$ , these mobiles can be considered as a continuum. Therefore, the upper sum can be approximated by an integral:

$$\sum_{m \in n} \beta \frac{g_{n,m}}{g_{j,m}} \approx \frac{N_n \beta}{S_n} \iint_{H_n} \frac{g_n(M)}{g_j(M)} dM \quad (\text{III.4})$$

where  $S_n$  is the surface of the cell determined by the BS  $n$ .

## 4. CAPACITY AND OPTIMIZATION

In this section we use the propositions 2 and 4 to determine the capacity of a network and the optimal transmitting powers.

### 4.1 Capacity of a network

In a regular hexagonal network (fig. 1), with  $d$  the minimal distance between two base stations, we have:  $S_n = \frac{\sqrt{3}}{2} d^2$

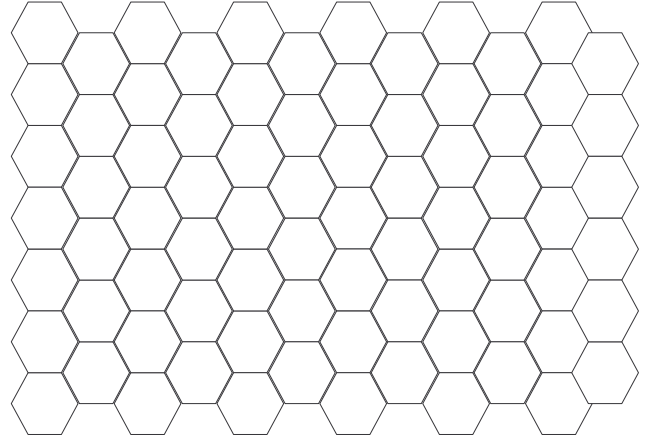


Figure 1: Hexagonal Network

We assume that, for all  $m, j$ ,  $g_j(M_m) = k O_j M_m^{-\eta}$ ;  $O_j$  being the position of BS  $j$ .

$$\sum_{m \in n} \beta \frac{g_{n,m}}{g_{j,m}} \approx \frac{2N_n \beta}{\sqrt{3} d^2} \iint_{H_n} \frac{O_n M^{-\eta}}{O_j M^{-\eta}} dM$$

Then, if the network is homogeneous, i.e. all the  $N_i$  are equal to  $N$ , we obtain:

$$\sum_m a_{mk} = N\beta(\alpha-1) + 2 \frac{N\beta}{\sqrt{3} d^2} \sum_{m,p} \iint_{H_0} \frac{(x^2 + y^2)^{\eta/2}}{\left[ \left( m \frac{\sqrt{3}}{2} + x \right)^2 + \left( p + \frac{m}{2} + y \right)^2 \right]^{\eta/2}} dx dy$$

(IV.1)

where  $H_0$  is the central hexagon, defined by

$$\begin{cases} -\frac{d}{2} \leq y \leq \frac{d}{2} \\ \frac{-d+|y|}{\sqrt{3}} \leq x \leq \frac{d-|y|}{\sqrt{3}} \end{cases}$$

The expression is homothetic invariant, *i.e.*  $d$ -independent and can be rewritten:

$$\sum_m a_{mk} = N\beta(\alpha-1) + 2\frac{N\beta}{\sqrt{3}} \sum_{m,p} \iint_H \frac{(x^2+y^2)^{\eta/2}}{\left[\left(m\frac{\sqrt{3}}{2}+x\right)^2 + \left(p+\frac{m}{2}+y\right)^2\right]^{\eta/2}} dx dy \quad (IV.2)$$

where  $H$  is defined by

$$\begin{cases} -\frac{1}{2} \leq y \leq \frac{1}{2} \\ \frac{-1+|y|}{\sqrt{3}} \leq x \leq \frac{1-|y|}{\sqrt{3}} \end{cases}$$

As a result of propositions 2 and 4, a sufficient condition for the system having a solution for all  $B$  is:

$$\sum_m a_{mk} < 1.$$

Besides, in a uniform and homogeneous hexagonal network,  $\sum_m a_{mk}$  does not depend upon  $k$ , because all columns are made of the same elements in a different order. Therefore:

$${}^t A \begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix} = \sum_m a_{mk} \begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix}, \text{ which proves that } \sum_m a_{mk} \text{ is an}$$

eigenvalue of  $A$ , and thus, equals the spectral radius of  $A$ . Therefore, and as a result of the remark on proposition 2,  $\sum_m a_{mk} < 1$  is a necessary and sufficient condition for the

system to have a solution for all  $B$ .

Therefore, in a uniform and homogeneous hexagonal network, the system has a solution if and only if:

$$N < \frac{1}{\beta(\alpha-1) + 2\frac{\beta}{\sqrt{3}} \sum_{m,p} \iint_H \frac{(x^2+y^2)^{\eta/2}}{\left[\left(m\frac{\sqrt{3}}{2}+x\right)^2 + \left(p+\frac{m}{2}+y\right)^2\right]^{\eta/2}} dx dy} \quad (IV.3)$$

## 4.2 Optimization of transmitting powers

The condition (IV.3) enables to determine the maximum number of mobiles per cell the network can stand. This number may be seen as an average number in a homogeneous network, and therefore, may be a non-integer number.

If this condition is met, then the optimal solution is, as shown in proposition 2:

$$P = (I + A + A^2 + \dots + A^k + \dots)B \quad (IV.4)$$

## 4.3 Numerical application

We consider  $\alpha=0.7$  and a voice service with a *SIR* target  $\gamma = -16dB$ . Equation (II.2) gives  $\beta \approx 0.02$ . We observe (fig. 2) an influence of the pathloss parameter  $\eta$  on the capacity of a cell in a regular homogeneous network, under the assumption that mobiles are uniformly distributed in each cell: as  $\eta$  increases, the average number of active mobiles supported per cell increases. The exponential parameter  $\eta$  can characterize the type of environment. In free space  $\eta=2$ , and in an urban environment, since there are many obstacles  $\eta$  increases. Though in a real network, the parameter  $\eta$  varies from 2.5 to 5, we show the capacity variations until  $\eta = 20$  to observe the theoretical limits of the capacity. As a result of (IV.3), maximal capacity tends to

$\frac{1}{\alpha\beta} \approx 71$  active mobiles per cell when  $\eta$  tends to infinity. The

increase of the capacity with the parameter  $\eta$  means that an environment with obstacles plays a role of 'protection' from the interferences: A mobile receives less interference power from the other base stations of the network. Thus it needs less transmitting power from its serving BS. As a consequence this last one can serve more mobiles.

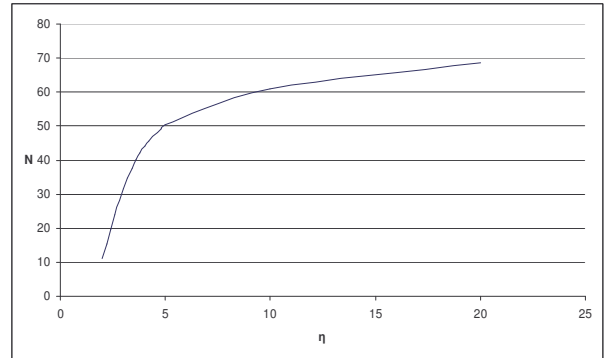


Figure 2: Influence of  $\eta$  on the capacity

When the average number of active mobiles per cell tends to the bound determined in (IV.3), the spectral radius of  $A$  tends to 1, and the solution  $P$  tends to infinity.

### Remark

In a real network, the base stations transmitting powers are limited to a value  $P_{max}$ : all elements of vector  $P$  are limited to a

fixed value  $P_{\max}$ . Moreover the power dedicated to the common channels can be considered as identical for all the base stations:  $P_{j,CCH} = P_0$ . As a result of the expression of vector B, all elements of B are bounded by  $P_0$ , as long as the thermal noise  $N_{th}$  can be considered negligible. Measurements in a real network showed that last assumptions are true for cell's size less than about 1 km. Let call  $N_0$  the right member of (IV.3). Then, the spectral radius of A is  $\frac{N}{N_0}$ .

The condition on vector P can be expressed  $\frac{P_0}{1 - N/N_0} \leq P_{\max}$  which can be rewritten as:

$$N \leq N_0 \left( 1 - \frac{P_0}{P_{\max}} \right) \quad (IV.5)$$

## 5. NETWORK DENSIFICATION

To answer an increasing traffic, we assume the provider adds base stations. We propose a densification (fig.3) of the hexagonal network by adding 6 new base stations around an existing base station, named  $BS_0$ . We suppose the distance between  $BS_0$  and anyone of the new base stations is  $d/2$ .

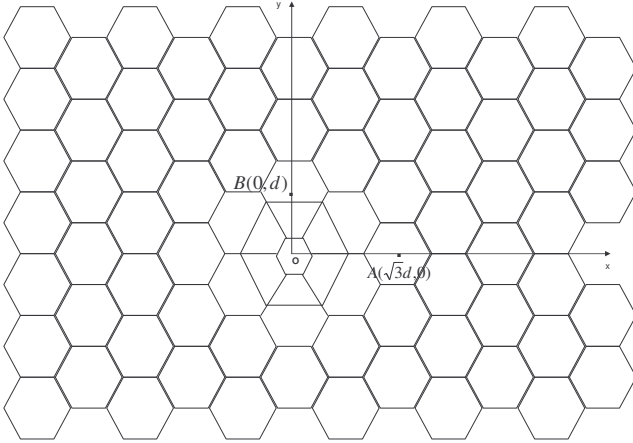


Figure 3: Densified hexagonal network

### 5.1 Analysis

If the cells of this network are defined by the perpendicular bisectors of the base stations, then we have 4 types of cells:

- 1 hexagon around  $BS_0$ , which area is  $S_1 = \frac{\sqrt{3}}{8} d^2$ ;

- 6 trapezes around the 6 new base stations, the area of each of them being  $S_2 = \frac{\sqrt{3}}{6} d^2$ ;
- 6 truncated hexagons adjacent to these 6 trapezes, the area of each of them being  $S_3 = \frac{19\sqrt{3}}{48} d^2$ ;
- An infinity of hexagons identical to those studied in the first application, the area of each of them being  $S_4 = \frac{\sqrt{3}}{2} d^2$ .

Let's call these 4 types of cells respectively  $H_1, H'_2, H'_3, H_4$ . We call  $N_i$  the number of active mobiles in cell  $H_i$ ,  $N_2$  the average number of active mobiles in  $H'_2$  type cells,  $N_3$  the average number of active mobiles in  $H'_3$  type cells, and  $N_4$  the average number of active mobiles in  $H_4$  type cells. As  $N_i$  are average numbers, they may be non-integer numbers. We assume, as in the first application, that the network is homogeneous. Therefore, we can consider that all cells of a certain type contain the average number of active mobiles corresponding to this type.

As in the first application, we assume that, for all  $m, j$ ,  $g_j(M_m) = k O_j M_m^\eta$ ;  $O_j$  being the position of BS  $j$ .

The condition  $\sum_m a_{mk} < 1$  must be verified for all  $k$ . If we assume that mobiles are uniformly distributed in each cell, the condition is expressed:

For all  $j$ ,

$$N_j \alpha \beta + \sum_{n \neq j} \frac{N_n \beta}{S_n} \iint_{H_i} \frac{O_i M^\eta}{O_j M^\eta} dM < 1 \quad (V.1)$$

When  $k$  is in a  $H_4$  type hexagon situated far away from  $BS_0$ , then the effects of the new base stations can be neglected, and the neighborhood of  $k$  is similar to the first application. Therefore,  $N_4$  must verify:

$$N_4 < \frac{1}{\beta(\alpha-1)+2 \frac{\beta}{\sqrt{3}} \sum_{m,p} \iint_{H_i} \frac{(x^2+y^2)^{\eta/2}}{\left[ \left( \frac{m\sqrt{3}}{2} + x \right)^2 + \left( p + \frac{m}{2} + y \right)^2 \right]^{\eta/2}} dx dy} \quad (V.2)$$

### 5.2 Numerical application

We now explicit (V.1) for  $k$  being respectively in  $H_1$ , a  $H'_2$  type trapeze, a  $H'_3$  type truncated hexagon, and finally a  $H_4$  type hexagon centered in  $A(\sqrt{3}d, 0)$ . Strictly speaking, the condition should be tested for all cells. However, if it is verified for these 4 types of cells, added to condition (V.2), one can reasonably assume it is verified everywhere.

After calculation, we obtain the following numerical inequalities for  $\eta = 3$ :

if k is in  $H_1$ :

$$0.014N_1 + 0.0113N_2 + 0.00398N_3 + 0.006N_4 < 1$$

if k is in a  $H_2$  type trapeze:

$$0.0018N_1 + 0.0203N_2 + 0.00724N_3 + 0.00684N_4 < 1$$

if k is in a  $H_3$  type truncated hexagon :

$$0.000157N_1 + 0.00487N_2 + 0.0171N_3 + 0.0113N_4 < 1$$

if k is in a  $H_4$  type hexagon centered in  $A(\sqrt{3}d, 0)$ :

$$0.000028N_1 + 0.00053N_2 + 0.0044N_3 + 0.0272N_4 < 1$$

Besides, the condition on  $N_4$  writes, for  $\eta = 3$ :

$$N_4 < 31.6$$

Resolving the system of inequalities leads to the following bounds, maximizing the total number of active mobiles:

$$N_1 < 28.88$$

$$N_2 < 25.66$$

$$N_3 < 28.88$$

$$N_4 < 31.6$$

Therefore, by adding 6 new cells, we enable the network to stand  $28.88+6*25.66+6*28.88=356.12$ , rounded to 356 active mobiles in an area previously covered by 7 hexagons, instead of  $7*31.6=221.2$  before. The densification enables about 135 more active mobiles to enter the network in an area of  $7\frac{\sqrt{3}}{2}d^2$ .

## 6. CONCLUSION

We developed an analytical approach of cellular networks, considering the distribution of mobiles as a continuum. We obtained explicit expressions of a cell capacity and optimized the

base stations transmitting powers. Our analytical model enables to know the precise influence of a mobile on a given zone of a network whatever its position. As a numerical example, we calculated the maximum number of mobiles per cell in a homogeneous network and we showed how to optimize the densification. We moreover showed the role played by the environment on the capacity.

Our approach can be used for any frequency reuse 1 network and any attenuation model. As a consequence, the application example we studied, focused on CDMA networks, can be used for networks based on other technologies, such as OFDMA.

## 7. REFERENCES

- [1] A. J. Viterbi, CDMA Principles of Spread Spectrum Communication, Wesley, 1995.
- [2] T. Bonald and A. Proutiere, Wireless Downlink Data Channels: User Performance and Cell Dimensioning, ACM Mobicom 2003
- [3] Jaana Laiho, Achim Wacker Tomas Novosad, "Radio network planning and optimisation for UMTS"
- [4] Hiltunen, K., De Bernardi, R. WCDMA Downlink capacity estimation, VTC 2000, p. 992-996
- [5] F. Baccelli, B. Błaszczyszyn, and F. Tournois (2003) Downlink admission/congestion control and maximal load in CDMA networks, in Proc. of IEEE INFOCOM'03
- [6] F. Baccelli, B. Błaszczyszyn, and F. Tournois. Stochastic capacity of large CDMA networks., INRIA Report, 2002.
- [7] K.S. Gilhousen, I.M. Jacobs, R. Padovani, A.J. Viterbi, L.A. Weaver, and Ch.E. Wheatley III. On the capacity of a cellular CDMA system. IEEE Trans. Veh. Technol., 40:303–312, 1991.
- [8] R.W. Nettleton and H Alavi. Power control for spread spectrum cellular mobile radio system. Proc. IEEE Trans. Veh. Technol.Conf., pages 242–246, 1983
- [9] Neelesh Mehta, Andreas Molisch, Larry Greenstein Orthogonality Factor in WCDMA Downlinks in Urban MacrocellularEnvironments, Proceedings Globecom 2005