Polarization Diversity in Ring Topology Networks

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***Abstract*—Polarization diversity is generally achieved by using two orthogonal polarizations on the same link, which enables to double the available bandwidth. In this paper, we study the possibility to connect the nodes of a line-of-sight ring topology network with one single channel for all the links, with the condition that the polarization of any link is orthogonal to the polarization of the two adjacent links. The solution proposed in this paper can improve spectrum efficiency by up to 50% in comparison with the widespread polarization multiplexing solution.**

I. Introduction

Polarization diversity is regarded as an efficient mean to meet the growing demand for wireless spectrum in line-of-sight links [1], [2]. In most cases, this solution is restricted to horizontal/vertical or $\pm 45°$. This enables to double the spectral efficiency on a single channel [3]. To the best of our knowledge, network coverage with one single channel by just optimizing the polarization directions has not been studied yet. In this paper, we show how an appropriate use of polarization diversity in ring topology networks improves spectrum efficiency.

While the current approach focuses on optimizing a point-to-point link, we propose a paradigm shift for polarization diversity in line-of-sight networks: instead of restricting the choice to two pre-defined polarizations, our approach is based on inclined polarizations fulfilling the condition that the polarization of any link is orthogonal to the polarization of the two adjacent links. This approach is suitable for all kinds of ring topology wireless networks whether the base stations are fixed or mobile and can use any kind of polarized antenna. As it will be shown, the polarizations will be determined by the base stations' locations. Therefore, in a network with fixed base stations, our solution will not require any extra cost compared to networks using horizontal/vertical polarizations, while in a network with mobile base stations, a centralized control system will be required.

II. Polarization in Ring Networks

In a chain comprising the nodes A, B and C, the polarization $\overrightarrow{E_{AB}}$ between A and B may be chosen arbitrarily (upon the condition it is orthogonal to the line AB), but then, the reuse of the same frequency requires that the polarization $\overrightarrow{E_{BC}}$ between B and C shall be orthogonal to $\overrightarrow{E_{AB}}$. Since $\overrightarrow{E_{BC}}$ shall also be orthogonal to the line BC, the direction of $\overrightarrow{E_{AB}}$ generally determines that of $\overrightarrow{E_{BC}}$. This solution can be extended to any number of nodes, provided that all links are line-of-sight. Since the polarization of a link generally determines the polarization of the next one, the use of one single frequency by polarization diversity in a ring topology network faces the question of closing the loop: the first polarization will determine the second one, which will determine the third one and so forth until the last one. The last polarization, which is determined by a sequence of constraints, is not necessarily orthogonal to the first one. In this section, we will study under what conditions it is possible to choose the first polarization in such a way that this orthogonality property is fulfilled.

A. Formulating the problem

Let us consider n nodes A_1, A_2,..., A_n. We define the following unitary vectors:
$\overrightarrow{u_1} = \frac{\overrightarrow{A_1A_2}}{\left\|\overrightarrow{A_1A_2}\right\|}, \overrightarrow{u_2} = \frac{\overrightarrow{A_2A_3}}{\left\|\overrightarrow{A_2A_3}\right\|}, \ldots, \overrightarrow{u_n} = \frac{\overrightarrow{A_nA_1}}{\left\|\overrightarrow{A_nA_1}\right\|}$
and the polarizations: $\overrightarrow{E_1}$ between A_1 and A_2, $\overrightarrow{E_2}$ between A_2 and A_3,..., $\overrightarrow{E_n}$ between A_n and A_1,

Since each polarization shall be orthogonal to the previous one and to the propagation line, it must be parallel to the vector product of these two vectors:
$\overrightarrow{E_1}||\overrightarrow{E_n} \times \overrightarrow{u_1}, \quad \overrightarrow{E_2}||\overrightarrow{E_1} \times \overrightarrow{u_2}, \quad \ldots \quad, \overrightarrow{E_n}||\overrightarrow{E_{n-1}} \times \overrightarrow{u_n}$
Therefore, $\overrightarrow{E_n}$ must fulfill the following condition:

$$\overrightarrow{E_n}||((\overrightarrow{E_n} \times \overrightarrow{u_1}) \times \ldots \times \overrightarrow{u_{n-1}}) \times \overrightarrow{u_n} \quad (1)$$

Let us define n vectorial planes: P_1 orthogonal to $\overrightarrow{u_1}$, P_2 orthogonal to $\overrightarrow{u_2}$,..., P_n orthogonal to $\overrightarrow{u_n}$. We can now define the following n homomorphisms (for simplicity of notation, we put $P_0 = P_n$):

$$\phi_i : P_{i-1} \to P_i$$
$$\overrightarrow{x} \mapsto \overrightarrow{x} \times \overrightarrow{u_i}$$

Then, $\phi = \phi_n \phi_{n-1} \ldots \phi_1$ is an endomorphism in P_n.

The above condition (1) can be written:

$$\overrightarrow{E_n}||\phi(\overrightarrow{E_n}) \quad (2)$$

Therefore, the problem can be expressed as the search of an eigenvector for the endomorphism ϕ.

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It is well known that the eigenvalues of an endomorphism ϕ in a vectorial plane are the solutions of the equation:

$$X^2 - tr(\phi)X + det(\phi) = 0 \quad (3)$$

where $tr(\phi)$ and $det(\phi)$ are the trace and the determinant of ϕ, respectively.

This equation has solutions if and only if:

$$tr(\phi)^2 - 4det(\phi) \geq 0 \quad (4)$$

For each vectorial plane P_i, we define two vectors $\overrightarrow{v_i}$ and $\overrightarrow{w_i}$ fulfilling the following conditions:

- $\overrightarrow{v_i}$ is in the plane including A_i, A_{i+1} and A_{i+2} (for simplicity of notation, we put $A_{n+1} = A_1$ and $A_{n+2} = A_2$).
- ($\overrightarrow{u_i}$,$\overrightarrow{v_i}$,$\overrightarrow{w_i}$) is a direct orthonormal basis of the space.

Note that the choice of $\overrightarrow{v_i}$ is not unique: two opposite vectors fulfill the condition. Anyone of both can be chosen arbitrarily.

We also define in P_i two vectors $\overrightarrow{v_i'}$ and $\overrightarrow{w_i'}$ fulfilling the following conditions:

- $\overrightarrow{w_i'} = \overrightarrow{w_{i-1}}$ (for simplicity of notation, we put $\overrightarrow{w_0} = \overrightarrow{w_n}$).
- ($\overrightarrow{u_i}$,$\overrightarrow{v_i'}$,$\overrightarrow{w_i'}$) is a direct orthonormal basis of the space.

B_i=($\overrightarrow{v_i}$,$\overrightarrow{w_i}$) and B_i'=($\overrightarrow{v_i'}$,$\overrightarrow{w_i'}$) are two orthonormal bases of P_i. Since ($\overrightarrow{u_i}$,$\overrightarrow{v_i}$,$\overrightarrow{w_i}$) and ($\overrightarrow{u_i}$,$\overrightarrow{v_i'}$,$\overrightarrow{w_i'}$) are both direct orthonormal bases of the space, the change of basis matrix from B_i to B_i' is a rotation matrix:

$$R_i = \begin{pmatrix} \cos\alpha_i & -\sin\alpha_i \\ \sin\alpha_i & \cos\alpha_i \end{pmatrix} \quad (5)$$

We define the angle $\theta_i = \angle A_{i-1}A_iA_{i+1}$.

Since $\overrightarrow{v_{i-1}} \times \overrightarrow{u_i} = \cos\theta_i \overrightarrow{w_i'}$ and $\overrightarrow{w_{i-1}} \times \overrightarrow{u_i} = \overrightarrow{v_i'}$, the matrix of ϕ_i with respect to the bases B_{i-1} and B_i' is:

$$M_i = \begin{pmatrix} 0 & 1 \\ \cos\theta_i & 0 \end{pmatrix} \quad (6)$$

Therefore, the matrix of ϕ in B_n is:

$$M = R_n M_n R_{n-1} M_{n-1} \dots R_2 M_2 R_1 M_1 \quad (7)$$

B. Solution of the problem

1) All the nodes are in the same plane:
If all the nodes are in the same plane, it is possible to choose B_i and B_i' so that $B_i = B_i'$ for all i. Then, all the rotation matrices equal the identity matrix. Equation (7) becomes:

$$M = M_n M_{n-1} \dots M_2 M_1 \quad (8)$$

If n is an even number, equations (6) and (8) give:

$$M = \begin{pmatrix} \cos\theta_1 \cos\theta_3 \dots \cos\theta_{n-1} & 0 \\ 0 & \cos\theta_2 \cos\theta_4 \dots \cos\theta_n \end{pmatrix} \quad (9)$$

In this case, the eigenvectors are obviously $\overrightarrow{v_n}$ and $\overrightarrow{w_n}$.

If n is an odd number, equations (6) and (8) give:

$$M = \begin{pmatrix} 0 & \cos\theta_2 \cos\theta_4 \dots \cos\theta_{n-1} \\ \cos\theta_1 \cos\theta_3 \dots \cos\theta_n & 0 \end{pmatrix} \quad (10)$$

Then, $tr(\phi) = tr(M) = 0$,
and $det(\phi) = det(M) = -\cos\theta_1 \cos\theta_2 \dots \cos\theta_n$.
Equation (3) becomes:

$$X^2 - \cos\theta_1 \cos\theta_2 \dots \cos\theta_n = 0 \quad (11)$$

This equation has a solution if and only if $\cos\theta_1 \cos\theta_2 \dots \cos\theta_n \geq 0$, which means if and only if the number of acute angles in the polygon is odd.

2) General case:
In the general case, the calculation of M and $tr(\phi)$ is much more complicated. However since the determinants of rotation matrices R_i equal to 1, we can still calculate $det(\phi)$:

$$det(\phi) = det(M) = (-1)^n \cos\theta_1 \cos\theta_2 \dots \cos\theta_n \quad (12)$$

Therefore, if the number of acute angles in the polygon is odd, then $det(\phi) \leq 0$ and equation (3) has a solution.

The results are summarized in TABLE I.

Nodes	Odd number	Even number
In the same plane	Solution if and only if there is an odd number of acute angles.	There is a solution.
Not in the same plane	If the number of acute angles is odd, then there is a solution.	

TABLE I
ONE SINGLE FREQUENCY IN A RING NETWORK.

The best illustration of the advantages of the solution proposed in this paper over the present state of the art is the case of a triangle with three acute angles: while horizontal/vertical polarization enables to allocate two thirds of the available bandwidth to each link, our solution enables to allocate the full bandwidth to each link. In this case, the spectral efficiency is improved by 50%.

More generally, for any odd number n, in an n-sided polygon, the use of horizontal/vertical polarization enables to allocate at best $\frac{n-1}{n}B$ to each link, B being the available bandwidth. When our solution is applicable, it enables to allocate the whole bandwidth to each link.

III. Conclusion

Polarization diversity enables significant improvements regarding frequency allocation in various ring topology networks. The solution we propose in this paper can improve spectrum efficiency by up to 50% in the best case in comparison with horizontal/vertical polarization.

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