Optimal Transport Theory for Effective UAV-Based Wireless Communications under Flight Time Constraints

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I. Introduction and Motivation

The use of unmanned aerial vehicles (UAVs) as flying based stations has emerged as a key approach for boosting the coverage and capacity of existing wireless cellular or ad hoc networks. UAV-based wireless communications can effectively provide fast, reliable and cost-effective connectivity to areas which are either congested (e.g. hotspots) or poorly covered by terrestrial networks [1]–[3]. Despite the several benefits and real applications of using UAVs as aerial base stations, one must address many technical challenges such as performance analysis, deployment, user association, and flight time optimization.

In particular, the flight time duration of the UAVs presents a unique design challenge for UAV-based communication systems [4]. More specifically, the performance of such systems significantly depends on the *hover time* of each UAV, which is defined as the flight time during which the UAV must stay over a given area for providing wireless service to ground users. In fact, with a higher hover time of the UAV, the ground users can receive wireless service for a longer period. Thus, by increasing the hover time, the UAV is able to meet higher load requirements and serve a larger area. However, the hover time of the UAV is naturally limited due to the insufficient battery-provided, on-board energy. Hence, while analyzing the UAV-based communication systems, the hover time constrains of the UAVs must be taken into account.

The main contribution of this work is to propose a novel framework for optimized UAV-to-ground communications while considering the UAV's hover time constraints. In particular, we consider a scenario in which a single UAV is used to serve ground users with any arbitrary spatial distribution over the target area. Here, given the maximum possible hover time of the UAV at some pre-defined locations known as *stop points*, we maximize the average amount of data transmitted (service) to the users by finding optimal cell partitions associated to the stop points. In this case, using the powerful mathematical framework of *optimal transport theory* [5], we optimally partition the geographical area based on the users' distribution, hover time at each stop point, and locations of the flying UAV. Our results show the tradeoff between total amount of service and fairness among the users. Moreover, our proposed cell partitioning approach leads to a significantly higher fairness among the users compared to the weighted Voronoi diagram, while having similar performance in terms of total service.

II. System Model

Consider a geographical area $\mathcal{D} \subset \mathbb{R}^2$ within which a number of wireless users are spatially distributed following a given distribution $f(x,y)$ in the two-dimensional plane. Within this area, as shown in Figure 1, a single UAV is used as an aerial base station in order to provide wireless service for the ground users. In this model, we consider a set $\mathcal{M}$ of M stop points at which the flying UAV stops and serves the ground users. Let $\boldsymbol{s}_i = (x_i, y_i, h_i)$ be the three-dimensional coordinate of each stop point $i \in \mathcal{M}$. At each stop point i, h_i corresponds to the altitude of the UAV. We consider a downlink scenario in which the UAV, at each stop point, adopts a frequency division multiple access (FDMA) technique to service the ground users. The total bandwidth and transmit power of the UAV are denoted by B and P_t. Moreover, we use A_i to represent the area (cell) partition in which the ground users are served by the UAV located at stop point i. Hence, the geographical area is divided into M disjoint partitions each of which is associated with one of the stop points. Note that, in our model, all the users located at each cell partition A_i will be served simultaneously. Let τ_i be the *hover time* of the UAV located at stop point i. The hover time at each stop point is the time duration that the UAV must spend for servicing the ground users located at the corresponding cell partition. Clearly, if a given UAV can use a larger hover time, then it can provide more service time to its users. Hence, the amount of service to each user depends on two resources, the bandwidth allocated to the user and the hover time of the UAV. Clearly, the throughput of a user located at (x,y) connecting to the UAV at stop point i is given by:

$$C_i(x,y) = B(x,y)\log_2\left(1+\gamma_i(x,y)\right), \tag{1}$$

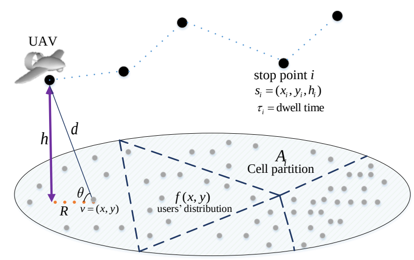


Fig. 1: Network model.

where $B(x,y)$ is the bandwidth allocated to the user at (x,y), and $\gamma_i(x,y)$ is the received SNR. Subsequently, the total service for the user provided by the UAV will be $L_i(x,y) = \tau_i C_i(x,y)$, where τ_i is the hover time, and $L_i(x,y)$ is the total number of bits transmitted to the user located at (x,y). Note that, the total service offered to the ground users depends on a number of key parameters such as the locations of the users and the stop points, the bandwidth allocated to the users, the hover times, and the cell partitions associated to the stop points.

Given this model, our goal is to maximize the average amount of service to the users under a fair resource (bandwidth and hover time) sharing policy by optimal partitioning of the area. In this case, the geographical area is optimally partitioned based on the hover time of the UAV as well as the spatial distribution of users.

III. Optimal Cell Partitioning

Here, we find the optimal cell partitions for which the average service, under a fair resource allocation policy, is maximized. In this case, each cell partition is assigned to one stop point in order to be serviced by the UAV. Clearly, the optimal cell partitions depend on the hover time at each stop point, and the spatial distribution of the users. Let τ_i be the hover time at stop point i in which the UAV provides service for the users in the corresponding cell partition, A_i. Under a fair time and frequency allocation policy, we have:

$$\frac{\tau_i B}{\int_{A_i} f(x,y)\mathrm{d}x\mathrm{d}y} = \frac{\tau_j B}{\int_{A_j} f(x,y)\mathrm{d}x\mathrm{d}y}, \;\; \forall i \neq j \in \mathcal{M}, \tag{2}$$

Then, considering $\sum\limits_{k=1}^{M} \int_{A_k} f(x,y)\mathrm{d}x\mathrm{d}y = 1$, we have:

$$\int_{A_i} f(x,y)\mathrm{d}x\mathrm{d}y = \frac{\tau_i}{\sum\limits_{k=1}^{M} \tau_k}, \;\; \forall i \in \mathcal{M}, \tag{3}$$

Next, we present the proposed optimization problem that is used to maximize the average service by optimally partitioning the target area. The service maximization problem is given by:

$$\max_{A_i,\, i\in\mathcal{M}} \sum_{i=1}^{M} \int_{A_i} \log_2\left(1+\gamma_i(x,y)\right) f(x,y)\mathrm{d}x\mathrm{d}y, \tag{4}$$

$$\text{s.t.} \;\; \int_{A_i} f(x,y)\mathrm{d}x\mathrm{d}y = \frac{\tau_i}{\sum\limits_{k=1}^{M} \tau_k}, \;\; \forall i \in \mathcal{M}, \tag{5}$$

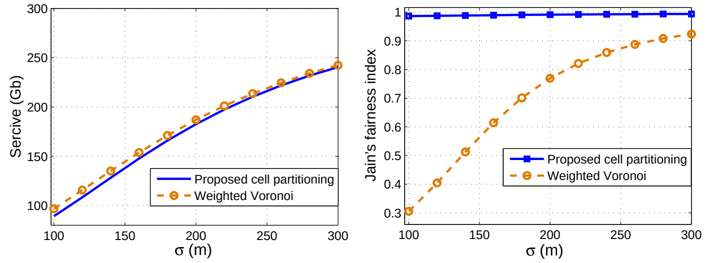
$$A_l \cap A_m = \emptyset, \;\; \forall l \neq m \in \mathcal{M}, \tag{6}$$

$$\bigcup_{i\in\mathcal{M}} A_i = \mathcal{D}, \tag{7}$$

where (5) is a constraint on the load of each cell partition. Also, (6) and (7) ensure that the cell partitions are disjoint and their union covers the entire target area $\mathcal{D}$.

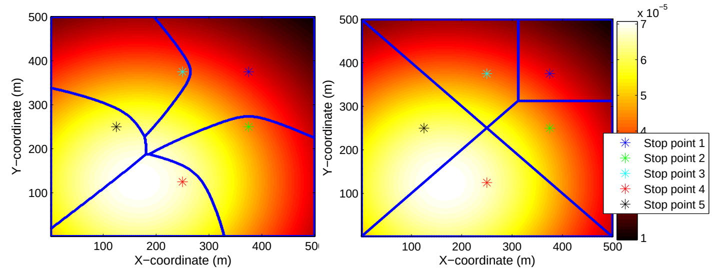
Solving (4) is challenging and intractable due to various reasons. First, the optimization variables $A_i, \forall i \in \mathcal{M}$, are sets of continuous mutually dependent partitions which must satisfy (5). Second, $f(x,y)$ can be any generic function of x and y that leads to the complexity of the given two-fold integrations. To solve the optimization problem in (4), we model the problem by exploiting optimal transport theory [5]. In particular, we first transform (4) into a semi-discrete optimal transport problem. Then, using the Kantorovich duality theorem [5], we propose an algorithm to find the optimal mapping between ground users and the stop points. Finally, given the optimal transport maps, we determine the optimal cell partitions corresponding to the target area. More details of our proposed solution are found in [6]. Next, we present our results obtained by solving the optimization problem in (4).

Fig. 2 compares the performance of our proposed cell partitioning with the classical weighted Voronoi partitioning in terms total service and fairness. In this case, we consider a truncated Gaussian distribution of users with a standard deviation σ and Jain's index for fairness. Fig. 2a shows that the average service to the users obtained by the proposed approach is very close



(a) Total service. (b) Fairness index.

Fig. 2: Proposed cell partitioning vs. weighted Voronoi diagram.



(a) Proposed optimal cells . (b) Weighted Voronoi .

Fig. 3: Cell partitions associated with stop points given the spatial distribution of users.

to the weighted Voronoi which is the optimal diagram in terms of the average service. In particular, for high values of σ the performance of our proposed approach coincides with the one for the weighted Voronoi diagram. From Fig. 2b, we can see that the Jain's index corresponding to the proposed cell partitioning method is above 0.96. However, in the weighted Voronoi case, it can decrease to 0.3 for a high non-uniform distribution of users. This is due to the fact that, in the Voronoi case, users located in highly congested partitions receive lower service than the partitions with low number of users. In the proposed approach, however, the resources (time and bandwidth) are fairly shared between the users thus leading to a higher fairness index.

Fig. 3, shows the proposed optimal cell partitions, obtained by solving (4), and weighted Voronoi for 5 stop points. In Fig. 3, areas shown by a lighter color have a higher population density. Fig. 3b shows that the cell partitions associated with stop points 4 and 5 have significantly more users than cell partition 1. Therefore, given the limited hover at time the stop points, users located at cell partitions 4 and 5 cannot be fairly served by UAVs. However, in the proposed optimal cell partitioning case, the cell partitions change such that the average service under a fair resource allocation constraint is maximized.

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