

ACHIEVABLE RATE MAXIMIZATION BY PASSIVE INTELLIGENT MIRRORS

Chongwen Huang¹, Alessio Zappone², Mérouane Debbah^{2,3}, Chau Yuen¹

1: Singapore University of Technology and Design (SUTD), 8 Somapah Rd, 487372, Singapore

2: Large Networks and Systems Group (LANEAS), Laboratoire des Signaux et Systèmes CentraleSupélec, CNRS, Univ. Paris Sud, Univ. Paris-Saclay, 91192 Gif-sur-Yvette, France

3: Mathematical and Algorithmic Sciences Laboratory, France Research Center, Huawei Technologies, Paris, France

ABSTRACT

This paper investigates the use of a Passive Intelligent Mirrors (PIM) to operate a multi-user MISO downlink communication. The transmit powers and the mirror reflection coefficients are designed for sum-rate maximization subject to individual QoS guarantees to the mobile users. The resulting problem is non-convex, and is tackled by combining alternating maximization with the majorization-minimization method. Numerical results show the merits of the proposed approach, and in particular that the use of PIM increases the system throughput by at least 40%, without requiring any additional energy consumption.

Index Terms— Multi-user MIMO, passive intelligent mirrors, non-convex optimization, majorization-minimization.

1. INTRODUCTION

The number of wireless devices is anticipated to reach 50 billions by 2020 [1–4]. This poses serious sustainability concerns for future cellular networks, which are demanded to provide $1000\times$ higher data rates compared to present networks, while at the same time halving energy consumptions [5–7]. This puts forth the critical needs for green and energy-efficient wireless solutions. The recent tutorial [8] and survey [9] provide a comprehensive review of green solutions, including renewable energy sources, energy-efficient hardware, and green radio resource allocation and transmission.

One recent technology that has a significant potential in reducing energy consumptions in wireless networks is the so-called Passive Intelligent Mirrors (PIM), i.e. a physical metasurface composed of many small-unit reflectors equipped with simple low-cost sensors and a cognitive engine. Each unit of the mirror is able to reflect a phase-shifted version of

an incoming electromagnetic field. Thus, by suitably designing the phase shifts, it is possible to constructively combine the signals reflected by the different units, which effectively makes the PIM an active medium. In other words, a PIM is able to act as an amplify-and-forward relay, but without requiring any dedicated energy source, thereby saving precious energy and enabling communication also in the presence of poor channel conditions. In addition, deployment costs are limited. A PIM can be easily integrated into the walls of the buildings surrounding the transmitter, as well as into ceilings of moving trains, laptop cases, and even on people's arms.

Previous research on PIM mainly focused on indoor scenarios [4, 10–13]. In [10, 11], the concept of intelligent wall was proposed, as a wall that can be equipped inside a building to improve indoor communications. In [12], a detailed analysis on the information transfer from the users to the large intelligent surfaces was carried out. However, these previous works did not provide any system design method, focusing merely on introducing the PIM idea in indoor scenarios.

In contrast, this work focuses on an outdoor scenario, considering a MISO downlink channel, and optimizing the transmit powers and the PIM phase shifts so as to maximize the system sum-rate. The resulting optimization problem is non-convex, and a provably convergent, low-complexity optimization method is developed merging the alternating optimization and majorization-minimization frameworks. Numerical results show that PIM can increase the system sum-rate by more than 40%, without any additional energy consumption.

2. SYSTEM MODEL

The considered system model is shown in Fig. 1. A base station (BS) equipped with M antennas serves K single-antenna users, through a PIM composed of N reflecting units embedded in a surrounding building, which acts a relay. The direct path between the BS and the mobile is neglected as it is assumed that no line-of-sight communication is present. Then, the discrete-time signal received at user k can be written as

$$y_k = \mathbf{h}_{k,2} \Theta \mathbf{H}_1 \mathbf{x} + w_k, k = 1, 2, \dots, K, \quad (1)$$

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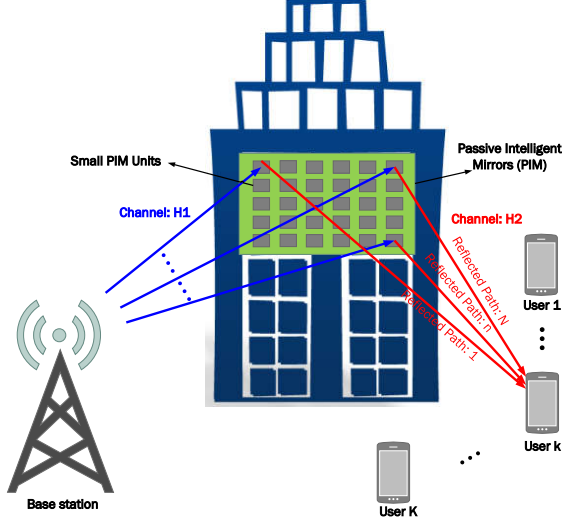


Fig. 1. A BS serves K mobile users through a PIM

where $\mathbf{h}_{k,2} \in \mathbb{C}^{1 \times N}$ and $\mathbf{H}_1 \in \mathbb{C}^{N \times M}$ denote the channel between the PIM and user k and between the BS and PIM, with entries modeled as independent and identically distributed (i.i.d.) complex circularly symmetric standard Gaussian variables; $\Theta = \text{diag}[e^{j\theta_1}, \dots, e^{j\theta_N}]$ is the PIM phase-shift matrix, with j the imaginary unit; $w_k \sim \mathcal{CN}(0, \sigma^2)$ models the thermal noise at user k ; $\mathbf{x} = \sum_{k=1}^K \sqrt{p_k} \mathbf{g}_k s_k$ is the BS transmit signal, with p_k , s_k , and \mathbf{g}_k the transmit power, information symbol, and beamforming vector intended for user k . Then, for all k , the SINR enjoyed by user k is

$$\gamma_k = \frac{p_k |\mathbf{h}_{2,k} \Theta \mathbf{H}_1 \mathbf{g}_k|^2}{\sum_{i=1, i \neq k}^K p_i |\mathbf{h}_{2,k} \Theta \mathbf{H}_1 \mathbf{g}_i|^2 + \sigma^2}. \quad (2)$$

2.1. Problem formulation

The goal of this work is to optimize the transmit powers and the matrix Θ for system sum-rate maximization. To make the problem tractable, we employ zero-forcing transmission, which is optimal in the high-SNR regime [14, 15]. Assuming $N \geq K$, and defining $\mathbf{G} = [\mathbf{g}_1, \dots, \mathbf{g}_K] \in \mathbb{C}^{M \times K}$ and $\mathbf{H}_2 = [\mathbf{h}_{2,1}^T, \dots, \mathbf{h}_{2,K}^T]^T \in \mathbb{C}^{K \times N}$, zero-forcing is achieved by setting $\mathbf{G} = (\mathbf{H}_2 \Theta \mathbf{H}_1)^+$, with $(\cdot)^+$ denoting pseudo-inversion. Then, defining the matrix $\mathbf{P} = \text{diag}[p_1, \dots, p_K]$ and denoting the maximum feasible transmit power at the BS by P_{max} , and the k -th user's rate requirement by $R_{min,k}$, the sum-rate maximization problem can be formulated as

$$\max_{\Theta, \mathbf{P}} \sum_{k=1}^K \log_2 \left(1 + \frac{p_k}{\sigma^2} \right) \quad (3a)$$

$$\text{s.t. } \log_2 \left(1 + \frac{p_k}{\sigma^2} \right) \geq R_{min,k}, \forall k = 1, \dots, K \quad (3b)$$

$$\text{tr}((\mathbf{H}_2 \Theta \mathbf{H}_1)^+ \mathbf{P} (\mathbf{H}_2 \Theta \mathbf{H}_1)^{+H}) \leq P_{max} \quad (3c)$$

$$0 \leq \theta_i \leq 2\pi, \forall i = 1, \dots, N \quad (3d)$$

Problem (3) is non-convex, and especially the optimization with respect to Θ appears challenging [16]. The next section introduces a computationally-affordable way to tackle (3).

3. PROPOSED APPROACH

In order to tackle (3) with affordable complexity, a convenient approach is to separately and iteratively optimize \mathbf{P} and Θ .

3.1. Optimization with respect to Θ

For fixed \mathbf{P} , Problem (3) becomes the feasibility test

$$\max_{\Theta} 1 \quad (4a)$$

$$\text{s.t. } \text{tr}((\mathbf{H}_2 \Theta \mathbf{H}_1)^+ \mathbf{P} (\mathbf{H}_2 \Theta \mathbf{H}_1)^{+H}) \leq P_{max} \quad (4b)$$

$$0 \leq \theta_i \leq 2\pi, \forall i = 1, \dots, N \quad (4c)$$

As a first step, it is convenient to apply the change of variable $\phi_k = e^{j\theta_k}$, which leads to the problem:

$$\max_{\Phi} 1 \quad (5a)$$

$$\text{s.t. } \text{tr}((\mathbf{H}_2 \Phi \mathbf{H}_1)^+ \mathbf{P} (\mathbf{H}_2 \Phi \mathbf{H}_1)^{+H}) \leq P_{max} \quad (5b)$$

$$|\phi_i| = 1, \forall i = 1, \dots, N, \quad (5c)$$

The challenge in solving Problem (5) lies in the fact that its objective is non-differentiable and that (5c) is a non-convex constraint. To proceed further, we observe that (5) is feasible if and only if the solution of the problem

$$\min_{\Phi} \text{tr}((\mathbf{H}_2 \Phi \mathbf{H}_1)^+ \mathbf{P} (\mathbf{H}_2 \Phi \mathbf{H}_1)^{+H}) \quad (6a)$$

$$\text{s.t. } |\phi_i| = 1, \forall i = 1, \dots, N, \quad (6b)$$

is such that the objective can be made lower than P_{max} . At this point, we observe that (6a) can be rewritten as follows:

$$\begin{aligned} \text{tr}((\mathbf{H}_2 \Phi \mathbf{H}_1)^+ \mathbf{P} (\mathbf{H}_2 \Phi \mathbf{H}_1)^{+H}) &= \|\mathbf{H}_1^+ \Phi^{-1} \mathbf{H}_2^+\|_F \\ &= \|\text{vec}(\mathbf{H}_1^+ \Phi^{-1} \mathbf{H}_2^+)\|^2 = \|(\mathbf{H}_2^+ \otimes \mathbf{H}_1^+) \text{vec}(\Phi^{-1})\|^2, \end{aligned} \quad (7)$$

where we have exploited the properties of the Frobenius matrix norm and the connection between the vectorization operator and the Kronecker product. As we will show, the objective in (7) enables to deal with the non-convex constraint (6b), provided one is able to reformulate (7) into a differentiable function. To this end, a convenient approach is to resort to the Majorization-Minimization (MM) method [17–19]. The MM method is an iterative approach that, in the i -th iteration, maximizes an upper-bound of (7). However, for any i -th iteration, the upper-bound maximized in i -th iteration and the true objective (7) must be equal when evaluated at the maximizer computed in $(i-1)$ -th iteration. The MM is attractive because it enjoys the monotonic improvement property, i.e., it monotonically decreases the value of the true objective (7) after each iteration. This also implies converges in the objective

value, since (7) is lower-bounded over the problem feasible set. Nevertheless, the challenge in using the MM method lies in determining a suitable upper bound of (7), which fulfills the theoretical requirements of the method, (i.e., coincides with the true objective at a given point), and is also easier to minimize compared with the original objective. For the case at hand, a convenient upper-bound is provided in the following lemma.

Lemma 1 Consider (6). Then, for any feasible $\mathbf{x} = \text{vec}(\Phi^{-1})$, and given any feasible point \mathbf{x}_t , a suitable upper-bound to employ the MM method is:

$$\|(\mathbf{H}_2^+ \otimes \mathbf{H}_1^+) \mathbf{x}\|^2 \leq f(\mathbf{x}|\mathbf{x}_t) = \mathbf{x}^H \mathbf{M} \mathbf{x} + 2\text{Re}(\mathbf{x}^H (\mathbf{L} - \mathbf{M}) \mathbf{x}_t) + \mathbf{x}_t^H (\mathbf{M} - \mathbf{L}) \mathbf{x}_t^H, \quad (8)$$

wherein, $\mathbf{M} = c_t^{max} \mathbf{A} \mathbf{A}^H$, $\mathbf{L} = \mathbf{A} (\text{diag}(\mathbf{c}_t) - N^2 \mathbf{I}) \mathbf{A}^H$, $\mathbf{A} = \mathbf{H}_2^+ \otimes \mathbf{H}_1^+$, $\mathbf{c}_t = |\mathbf{A}^H \mathbf{x}_t|$, and $c_t^{max} = \max(\mathbf{c}_t)$ is a maximum element in the vector \mathbf{c}_t .

Proof: The proof leverages the second-order Taylor expansion of (8). Full details are omitted due to space constraints.

Based on Lemma 1, in each iteration of the MM method we are faced by the problem

$$\min_{\mathbf{x}} f(\mathbf{x}|\mathbf{x}_t) \quad (9a)$$

$$\text{s.t. } |x_i| = 1, \forall i = 1, \dots, N. \quad (9b)$$

Proposition 1 For any \mathbf{x}_t , (9) is solved by $\mathbf{x} = e^{-j\arg(\mathbf{y})}$, with $\arg(\mathbf{y})$ denoting the component-wise phase of \mathbf{y} , and

$$\mathbf{y} = \frac{-\mathbf{A} (\text{diag}(\mathbf{c}_t) - c_t^{max} \mathbf{I} - N^2 \mathbf{I}) \mathbf{A}^H}{c_t^{max} \mathbf{A} \mathbf{A}^H} \mathbf{x}_t \quad (10)$$

Proof: The result follows from the analysis of the stationary points of (9a). Details are omitted due to space constraints.

3.2. Optimization with respect to P

For fixed Θ , Problem (3) is stated as,

$$\max_{\mathbf{P}} \sum_{k=1}^K \log_2 \left(1 + \frac{p_k}{\sigma^2} \right) \quad (11a)$$

$$\text{s.t. } p_k \geq \sigma^2 (2^{R_{min,k}} - 1), \forall k = 1, \dots, K \quad (11b)$$

$$\text{tr}((\mathbf{H}_2 \Theta \mathbf{H}_1)^+ \mathbf{P} (\mathbf{H}_2 \Theta \mathbf{H}_1)^{+H}) \leq P_{max} \quad (11c)$$

Problem (11) is convex and thus can be solved by means of standard convex optimization techniques [20, 21]. Specifically, analyzing the Karush–Kuhn–Tucker (KKT) optimality conditions of (11) yields the following closed-form expression for the solution of (11).

Lemma 2 Problem (11) admits the following solution

$$p_k = [\alpha \lambda_k - \sigma^2]^+ + \sigma^2 (2^{R_{min,k}} - 1) \lambda_k^{-1}, \quad (12)$$

where water level $\alpha = \frac{1}{q} (P_{max} - \sum_{k=1}^K \sigma^2 (2^{R_{min,k}} - 1) \lambda_k^{-1} + \sigma^2 \sum_{k=1}^q \lambda_k^{-1})$ is the Lagrange multiplier associated to (11c), λ_k is the k -th eigenvalue of $(\mathbf{H}_2 \Theta \mathbf{H}_1) (\mathbf{H}_2 \Theta \mathbf{H}_1)^H$, q is the number of non-zero eigenvalues λ_k , and $[x]^+$ denotes $\max(0, x)$.

Finally, the overall algorithm can be stated as in Algorithm 1, wherein the inner loop implements the MM algorithm, and the outer loop the alternating maximization. We remark that each iteration of Algorithm 1 monotonically increases the value of the overall problem objective in (3a). Thus, Algorithm 1 converges in the value of the objective, since (3a) is continuous over the compact feasible set of (3), and thus is upper-bounded.

Algorithm 1 Proposed Sum-rate Maximization Algorithm

- 1: **Require:** $P_{max}, \sigma^2, \{R_{min,k}\}_{k=1}^K, \mathbf{H}_2$, and \mathbf{H}_1 ;
 - 2: **Initialize** $\{p_k\}_{k=1}^K$, and set $t=0$;
 - 3: **Repeat:**
 - 4: **Repeat:**
 - 5: $\mathbf{c}_t = |\mathbf{A}^H \mathbf{x}_t|$; $c_t^{max} = \max(\mathbf{c}_t)$; $\mathbf{M} = c_t^{max} \mathbf{A} \mathbf{A}^H$;
 - 6: $\mathbf{L} = \mathbf{A} (\text{diag}(\mathbf{c}_t) - N^2 \mathbf{I}) \mathbf{A}^H$;
 - 7: $\mathbf{y} = \frac{-\mathbf{A} (\text{diag}(\mathbf{c}_t) - c_t^{max} \mathbf{I} - N^2 \mathbf{I}) \mathbf{A}^H}{c_t^{max} \mathbf{A} \mathbf{A}^H} \mathbf{x}_t$;
 - 8: $\mathbf{x}_{t+1} = e^{j\arg(\mathbf{y})}$; $t \leftarrow t + 1$;
 - 9: **Until** Convergence is reached; Obtain Φ ;
 - 10: **If** (6a) evaluated at Φ is lower than P_{max} ;
 - 11: $p_k = [\alpha \lambda_k - \sigma^2]^+ + \sigma^2 (2^{R_{min,k}} - 1) \lambda_k^{-1}$, for all k ;
 - 12: **Else**
 - 13: Break and declare unfeasibility.
 - 14: **Until** Convergence is reached; Φ and p_k are obtained.
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4. NUMERICAL RESULTS

In our numerical simulations, we considered the system scenario described in Section 2, with users randomly placed in an area of $625 m^2$ with channels generated as realizations of random matrices with i.i.d. entries drawn from a standard complex Gaussian distribution. All results have been obtained by averaging over 500 independent channels and users positions realizations. We define the Signal Noise Ratio (SNR), $\text{SNR} = P_{max} / \sigma^2$.

Fig. 2 compares the achievable sum rate versus SNR. We consider two sets of system parameters, namely $K = 16$, $M = 32$, $N = 32$ and $K = 8$, $M = 8$, $N = 8$, and the minimum QoS rate has been set to $R_{min,k} = R_{min} = \log_2(1 + \frac{\text{SNR}}{2K})$ bps/Hz for all $k = 1, \dots, K$ according to the different SNRs. The optimal solution of problem (3) is obtained through global optimization methods (Quasi-Newton search). This approach has an exponential complexity and is considered here only for benchmarking purpose. Resource allocation in a system without PIM is also considered as a baseline scheme. In this case only power allocation needs to

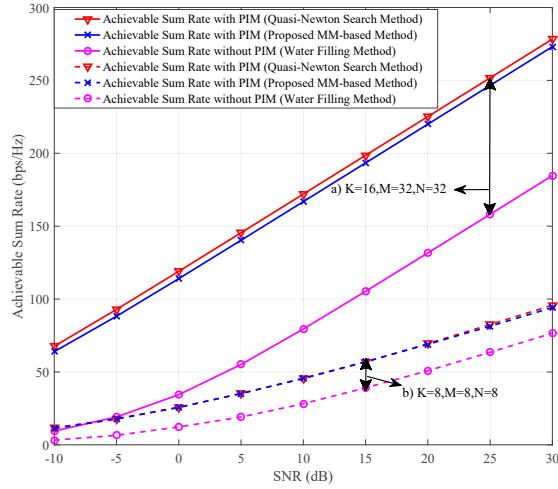


Fig. 2. Achievable rate versus SNR. $R_{min} = \log_2(1 + \frac{SNR}{2K})$ bps/Hz; a) $K = 16, M = 32, N = 32$; b) $K = 8, M = 8, N = 8$.

be performed, which is accomplished by the tools described in Section 3.2. We compare the achievable sum rate of the proposed Algorithm 1 with the optimal solution. It can be seen that employing the PIM increases the sum rate by more than 40%. Also, the gap becomes wider as the number of antennas and PIM units increases. Furthermore, Algorithm 1 suffers a limited gap compared to the global optimization method, which has a much higher complexity.

Fig. 3 shows the achievable sum spectral efficiency of the global optimization method and proposed Algorithm 1, versus the number of PIM units. The SNR is 20 dB, while $K = 16, M = 8, R_{min,k} = 2$ bps/Hz for all k . It can be seen that the proposed Algorithm 1 matches the sum spectral efficiency obtained by the global optimization method. Furthermore, this figure also shows that the more PIM units the larger the sum spectral efficiency in one cell, even though, as expected, the increase saturates as the number of PIM units grows.

Finally, Fig. 4 addresses the convergence speed of the MM-based method employed within Algorithm 1, in terms of the number of iterations to reach a given Mean Square Error (MSE) among two successive iterations, defined as

$$MSE = \frac{\|\Theta_{t+1} - \Theta_t\|^2}{\|\Theta_t\|^2} \quad (13)$$

System parameters have been set to $K = 16, M = 8$, and $N = 16; 32; 64$. It is seen that Algorithm 1 reaches acceptable values of MSE in a few dozens of iterations, which increases as N grows larger, since this corresponds to increasing the number of optimization variables. Nevertheless, recalling that each iteration of the MM method involves simple closed-form computations, Fig. 4 confirms the very limited complexity of the proposed MM-based method.

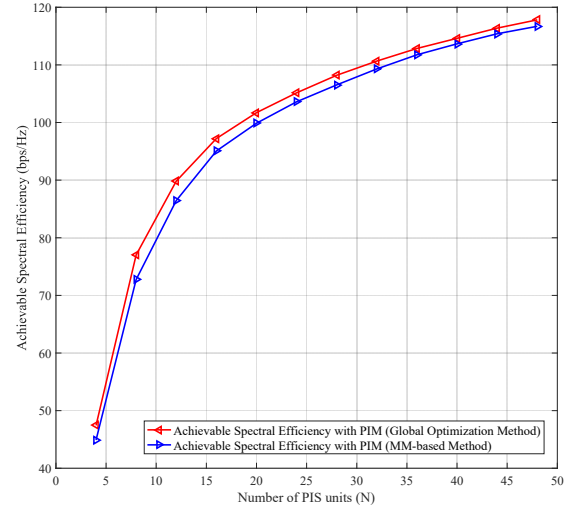


Fig. 3. Spectral Efficiency versus PIM units number. SNR = 20dB, $K = 16, M = 8, R_{min,k} = 2$ bps/Hz: a) Global optimization method; b) Proposed MM-based method.

5. CONCLUSION

A sum-rate maximization scheme for a PIM-based, multi-user MIMO system was developed. The non-convex radio resource allocation problem was tackled by combining MM and alternating optimization, to provide a provably convergent and low-complexity algorithm. Numerical result show that the proposed scheme achieves near-optimal performance, and improves by more than 40% the sum rate compared to traditional systems without PIM.

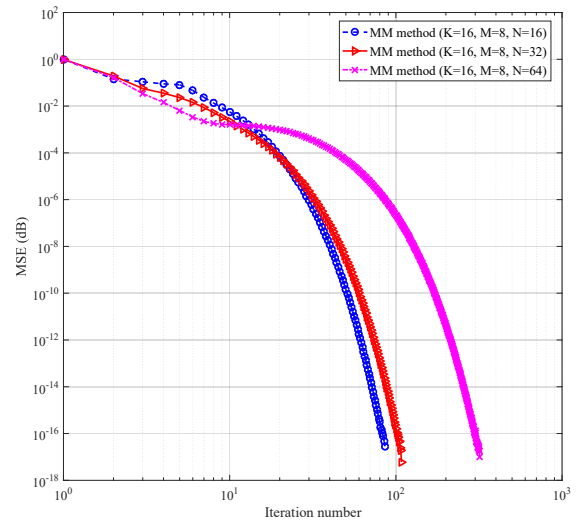


Fig. 4. MSE versus iteration number. $K = 16, M = 8, N = 16; 32; 64$.

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