Energy-Efficient Relay Assignment and Power Control in Multi-User and Multi-Relay Networks

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Abstract—We consider a multi-user, multi-relay network where each source communicates with its destination via an assigned relay. To ensure fairness among the users, the minimum of the users' energy efficiencies (in bit-per-Joule) is jointly maximized with respect to the relay assignment and transmit power of both source and relay nodes. The resulting algorithm is provably convergent, requires limited complexity, and significantly outperforms the baseline scenario in which no joint optimization of transmit powers and relay assignment is performed.

Index Terms—Energy efficiency, multi-user networks, multi-relay networks, relay assignment, power optimization.

I. Introduction

Radio resource allocation for energy efficiency has become a key performance indicator for the design of future mobile networks [1]. Furthermore, the use of relays constitutes a fundamental tool for extending the coverage of current and future wireless networks [2]. In the context of single- or multi-user networks, where multiple relays are available, properly designed relay assignment strategies have the potential of providing spatial diversity gains to the users, thus improving their performance [3]. The relay assignment problem is usually formulated to improve the error rate and throughput of the source-to-destination links, by assuming that their transmit powers are fixed, e.g. [4], [5]. The rise of energy efficiency as a key performance indicator for wireless networks, however, has put forth the relevance of investigating the relay assignment problem from an energy efficiency perspective.

The design of relay assignment schemes that aim at optimizing the energy efficiency is considered in several recent works focused on single-user networks [6]–[10] (and references therein). The goal is to minimize the transmit power of a multi-user orthogonal frequency-division multiplexing system. In [11], a power-allocation method that reduces the total power consumption while maintaining the required quality of service for application to single-user and multi-user relay systems is proposed. Other recent works study power allocation schemes for relay-aided networks, considering a fixed assignment. In

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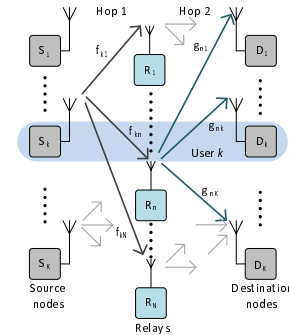
Fig. 1: Considered multi-user multi-relay network model.

[12], the energy efficiency of a multi-user and single-relay MIMO system is maximized. In [13], a relay-assisted cognitive system is considered. An information-theoretic analysis of the multi-way relay channel is available in [14]. Energy-efficient resource allocation policies for the multi-way relay channel are studied in [15]. In [16], schemes that minimize the energy expenditure of cloud radio relay-assisted networks under quality of service constraints are analyzed.

In the depicted context, the present work considers a multi-user and multi-relay network, and aims at *jointly* optimizing the relay assignment and transmit powers to maximize the minimum bit-per-Joule energy efficiency across the users, a problem that has not been considered in previous works. Indeed, as outlined above, most previous works focus only on either relay assignment or power control. Moreover, available works that jointly consider relay assignment and power control are focused on non-energy-efficient allocations, like [17], where a joint optimization of relay selection and resource allocation under rate constraints is considered.

The novel scenario considered in this work leads to a mixed-integer optimization problem, which is tackled by generalizing to the energy-efficient scenario the relay assignment strategies from [5], where SNR maximization was considered, and by employing generalized fractional programming tools to deal with the power allocation part of the problem [1].

II. System Model

We consider the dual-hop wireless relay network depicted in Fig. 1, where K source nodes $(S_1, \ldots, S_K)$ send information to their corresponding destination nodes $(D_1, \ldots, D_K)$ via N relay nodes $(R_1, \ldots, R_N)$ that operate either in decode-and-forward (DF) or in amplify-and-forward (AF) mode. Each node is equipped with a single antenna and operates in half-duplex mode. The transmit powers of the kth user and nth

relay, $n \in \{1, \ldots, N\}$, are p_k and q_n, respectively. Motivated by practical considerations, we consider the case in which each user is assisted by one and only one relay[1], thus assuming $N \geq K$. To avoid interference among the users, orthogonal (time, frequency) channels are used for transmission.

The fading channel between S_k and R_n and between R_n and D_k are denoted by $f_{k,n}$ and $g_{k,n}$, respectively. All channels are assumed to be independent and identically distributed, to remain constant during one transmission block, but to change independently over different transmission blocks. Moreover, the direct user-destination channels are assumed to be too weak for communication and hence are neglected. Then, the achievable rate, $r_{k,n}$, and bit-per-Joule energy efficiency, $\eta_{k,n}$, of the kth user-destination pair are:

$$r_{k,n} = \frac{1}{2K} \log_2 \left(1 + \gamma_{k,n} \left(p_k, q_n\right)\right) \text{ [bpcu]} \tag{1}$$

$$\eta_{k,n} = \frac{B \log_2 \left(1 + \gamma_{k,n} \left(p_k, q_n\right)\right)}{\lambda_k p_k + \mu_n q_n + P_{k,n}^{(c)}} \text{ [bit/Joule]} \tag{2}$$

where "bpcu" stands for bit per channel use, the pre-log factor $1/2K$ is due to the half-duplex operation mode and to the use of orthogonal transmission channels, λ_k and μ_n are the inverse of the power amplifier efficiencies of S_k and R_n, respectively, $P_{k,n}^{(c)}$ is the static power dissipated to operate the communication of user k through relay n, and $\gamma_{k,n}$ is the SNR of the $S_k - D_k$ pair through relay R_n, which depends on the particular relaying protocol being used. Let $\gamma_{k,n}^{(1)} = p_k \frac{|f_{k,n}|^2}{\sigma^2}$ and $\gamma_{k,n}^{(2)} = q_n \frac{|g_{k,n}|^2}{\sigma^2}$, be the SNRs of the $S_k - R_n$ and $R_n - D_k$ links, where σ^2 is the noise power. The SNRs enjoyed using the DF and AF relaying protocols are, respectively:

$$\gamma_{k,n}^{DF}(p_k, q_n) = \min\left(\gamma_{k,n}^{(1)}, \gamma_{k,n}^{(2)}\right) = \frac{\min\left(p_k |f_{k,n}|^2, q_n |g_{k,n}|^2\right)}{\sigma^2} \tag{3}$$

$$\gamma_{k,n}^{AF}(p_k, q_n) = \frac{\gamma_{k,n}^{(1)} \gamma_{k,n}^{(2)}}{\gamma_{k,n}^{(1)} + \gamma_{k,n}^{(2)} + 1} = \frac{p_k q_n |f_{k,n}|^2 |g_{k,n}|^2 / \sigma^2}{p_k |f_{k,n}|^2 + q_n |g_{k,n}|^2 + \sigma^2}. \tag{4}$$

Motivated by fairness considerations[2], our goal is to maximize the minimum energy efficiency across the users, with respect to $\{p_k\}_{k=1}^K$, $\{q_n\}_{n=1}^N$, and to the source-relay association $\{\delta_{k,n}\}_{k,n}$, where $\delta_{k,n} = 1$ if user k is assisted by relay n, and $\delta_{k,n} = 0$ otherwise.

III. PROBLEM FORMULATION AND SOLUTION

The energy efficiency of the kth user before knowing its associated relay is given by[3]:

$$\eta_k = \frac{\sum_{n=1}^N \log_2 \left(1 + \delta_{k,n} \gamma_{k,n} \left(p_k, q_n\right)\right)}{\lambda_k p_k + \sum_{n=1}^N \delta_{k,n} \left(\mu_n q_n + P_{k,n}^{(c)}\right)}. \tag{5}$$

Then, the maximization of the minimum energy efficiency among the users can be formulated as follows:

$$\max_{\{p_k, q_n, \delta_{k,n}\}_{k,n}} \min_{1 \leq k \leq K} \eta_k(p_k, q_n, \delta_{k,n}) \tag{6a}$$

$$\text{s.t. } p_k \in [0, p_{max,k}] , \ q_n \in [0, q_{max,n}] \ \forall k, n \tag{6b}$$

$$\delta_{k,n} \in \{0,1\}, \sum_{n=1}^N \delta_{k,n} = 1, \sum_{k=1}^K \delta_{k,n} \in \{0,1\} \ \forall k, n \tag{6c}$$

[1]Having multiple users associated to multiple relays would increase the system complexity and energy consumption [18], [19].

[2]If instead a system-wide performance optimization is desired, the system global energy efficiency is a more suitable performance metric.

[3]Recall that the users are assigned to only one relay.

where $p_{max,k}$ and $q_{max,n}$ are the maximum power levels of S_k and R_n, respectively, while (6c) ensures that each user is connected to one and only one relay.

The main challenges to solve (6) lie in its mixed-integer nature and in its non-concave fractional objective, which is not differentiable even with respect to only the transmit powers due to the $\min(\cdot)$ operator. We tackle these issues by using the alternating optimization method illustrated in Algorithm 1, alternatively optimizing the association variables $\{\delta_{k,n}\}_{k,n}$ for fixed transmit powers $\{p_k, q_n\}_{k,n}$, and the transmit powers for fixed association variables, until convergence.

Algorithm 1 Alternating maximization of the problem in (6)

Set $\{\bar{\delta}_{k,n}\}_{k,n} \in \{0,1\}^{KN}$;
repeat
  $\{\bar{p}_k, \bar{q}_n\}_{k,n} = \arg \max_{\{p_k, q_n\}_{k,n}} \min_{1 \leq k \leq K} \text{EE}_k(p_k, q_n, \bar{\delta}_{k,n})$,
  subject to (6b);
  $\{\bar{\delta}_{k,n}\}_{k,n} = \arg \max_{\{\delta_{k,n}\}_{k,n}} \min_{1 \leq k \leq K} \text{EE}_k(\bar{p}_k, \bar{q}_n, \delta_{k,n})$,
  subject to (6c);
until convergence

Algorithm 1 monotonically increases the value of the objective and converges in the value of the objective[4], even though global optimality cannot be theoretically guaranteed. The two optimization subproblems that constitute Algorithm 1 are discussed and solved in the next two sections.

A. Transmit Power Optimization

The first subproblem in Algorithm 1 assumes that the association variables $\delta_{k,n} = \bar{\delta}_{k,n}$ are fixed for all k, n, and optimizes the energy efficiency with respect to $\{p_k\}_k$ and $\{q_n\}_n$. Thus, the following optimization problem is considered:

$$\max_{\{p_k, q_n\}_{k,n}} \min_{1 \leq k \leq K} \eta_k(p_k, q_n) \tag{7a}$$

$$\text{s.t. } p_k \in [0, p_{max,k}] , \ [0, q_{max,n}] \ \forall k, n . \tag{7b}$$

The optimization problem in (7) is an instance of a generalized fractional program [20], which differs from the well-known class of fractional programs because of the presence of the $\min(\cdot)$ operator. An algorithm to globally solve generalized fractional programs was proposed in [20, Section 2.2] and named *generalized Dinkelbach's algorithm*. If each ratio within the $\min(\cdot)$ operator is the ratio of a concave function over a convex function, and all the constraints are convex, then the generalized Dinkelbach's algorithm finds the solution of (7) by solving a sequence of convex auxiliary problems that converge at a rate that is at least linear [20, Proposition 3.3]. As far as the specific problem at hand is concerned, the denominators in (7a) and the affine constraints in (7b) fulfill these requirements. Instead, as for the numerators in (7a), the analysis is more challenging. In particular, the DF and AF protocols need to be treated separately.

Proposition 3.1: Let the DF protocol be used. Every numerator in (7a) is jointly concave in p_k and q_n, for all k, n.

Proof: It needs to be proved that $\gamma_{k,n}$ is jointly concave in p_k and q_n, for all k, n. This follows since the logarithm of a concave function is concave and the sum of concave

[4]The objective (6a) is upper-bounded over the feasible set and thus it cannot increase indefinitely.

functions is concave. Also, $\gamma_{k,n}$ in (3) is the minimum of linear functions and, thus, it is concave, since the minimum of concave functions is concave. ∎

Proposition 3.2: With the AF protocol, every numerator in (7a) is jointly concave in p_k, q_n if $\gamma_{k,n}^{(1)}\gamma_{k,n}^{(2)} \geq \frac{1}{2}$ for all k,n.

Proof: Similar to DF, it needs to be proved that $\gamma_{k,n}$ is jointly concave in p_k and q_n, for all k,n. We observe that (4) can be equivalently studied as a function of $\gamma_{k,n}^{(1)}$ and $\gamma_{k,n}^{(2)}$, or as a function of p_k and q_n, since the two pairs of variables are related by a linear mapping that preserves concavity. We study the concavity of (4) as a function of $\gamma_{k,n}^{(1)}$ and $\gamma_{k,n}^{(2)}$, by computing its Hessian matrix. With some algebra, we obtain (8) shown at the top of the next page. The diagonal elements of (8) are non-positive, and, thus, it is negative semi-definite if its determinant is non-negative[5]. This is equivalent to:

$$2p_kq_n(p_k^2+2p_k+q_n^2+2q_n+2p_kq_n)-(p_k^2+2p_k+q_n^2+2q_n+1)$$
$$= (2p_kq_n-1)(p_k^2+2p_k+q_n^2+2q_n+2p_kq_n+1) \geq 0 \qquad (9)$$

from which the proof follows. ∎

Remark 1: The condition $\gamma_{k,n}^{(1)}\gamma_{k,n}^{(2)} \geq \frac{1}{2}$ in Proposition 3.2 is not very restrictive. The term $\gamma_{k,n}^{(1)}\gamma_{k,n}^{(2)}$, in fact, is the product of the SNRs at the relay and destination, which cannot be too low to ensure a reliable communication link.

B. Users-Relay Association Optimization

The second subproblem in Algorithm 1 assumes that the transmit powers $p_k = \bar{p}_k$ and $q_n = \bar{q}_n$ are fixed for all k,n, and optimizes the energy efficiency with respect to $\{\delta_{k,n}\}_{k,n}$. Thus, the following optimization problem is considered:

$$\max_{\{\delta_{k,n}\}_{k,n}} \min_{1 \leq k \leq K} \eta_k(\bar{p}_k, \bar{q}_n, \delta_{k,n}) \qquad (10a)$$

$$\text{s.t. } \delta_{k,n}\in\{0,1\}, \sum_{n=1}^N\delta_{k,n}=1, \sum_{k=1}^K\delta_{k,n}\in\{0,1\} \; \forall k,n. \qquad (10b)$$

The optimal solution of (10) can be found as follows. Since the transmit powers $\{\bar{p}_k\}_{k=1}^K$ and $\{\bar{q}_n\}_{n=1}^N$ are given, one can stack, for any possible relay assignment, the energy efficiencies of all the users in a $K \times N$ matrix $\mathcal{E} = \{\eta_{k,n}\}_{k,n}$, whose elements are $\eta_{k,n} = \frac{B\log_2(1+\gamma_{k,n}(\bar{p}_k,\bar{q}_n))}{\lambda_k\bar{p}_k+\mu_n\bar{q}_n+P_{k,n}^{(c)}}$ (see (2)). The kth row gives the energy efficiencies of the k if assisted by relay $\{R_n\}_{n=1}^N$, and the nth column gives the energy efficiencies of all the users if assisted by Relay R_n. Since a given relay can assist the transmission of only one user, a selection policy is admissible if only one entry of each row is non-zero, and if each of those entries is in a different column. Since the max-min problem in (10) considers individual performance and fairness, an optimal assignment can be obtained by generalizing the algorithms from [4], [5], to the energy efficiency maximization scenario.

In particular, we propose the following two-step algorithm.

Step I: The energy efficiency matrix $\mathcal{E}$ is computed as explained above. Algorithm 2 is applied to $\mathcal{E}$ to maximize the minimum energy efficiency across the users. Let u and R_u be the user and the associated relay that achieve the optimum.

Step II: After Step I, the uth row and R_uth column of $\mathcal{E}$ are deleted and Algorithm 2 is applied to the new matrix $\mathcal{E}_u$. This is iterated until all users are associated to a relay.

Both steps use Algorithm 2 to operate on a given energy efficiency matrix by the functions[6] `find_another_relay()` and `check_relay_availability()`, where the user associated to relay R_j is denoted by $\mathcal{U}(R_j)$ and its energy efficiency by $\eta(\mathcal{U}(R_j), R_j)$.

Algorithm 2 Optimal Relay Assignment for a Given Energy Efficiency Matrix

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1: If in Step I, make a random assignment for all users. If in Step II, use the previous selection
2: Set all the relays as "unmarked"
3: Find the smallest energy efficiency among the selection. Set the value η_m and relay index k_m
4: Check for a better assignment for the user $\mathcal{U}(R_{k_m})$ using the function `find_another_relay`$(\mathcal{U}(R_{k_m}), k_m, \eta_m)$
5: If a better assignment is found, update the relay assignment accordingly. Otherwise terminate the algorithm.

`find_another_relay`$(\mathcal{U}(R_j), j, \eta_m)$

1: For every "unmarked" relay R_n with $\eta(\mathcal{U}(R_j), R_n) > \eta_m$, do the following in non-increasing order of $\eta(\mathcal{U}(R_j), R_n)$
2: Run the function `check_relay_availability`(R_n, η_m)
3: **if** R_n is available **then**
4:   Remove relay R_j from user $\mathcal{U}(R_j)$
5:   Assign relay R_n to user $\mathcal{U}(R_j)$
6: **else**
7:   Continue on to next R_n and go to line 2
8: **end if**
9: If all relays are unavailable, $\mathcal{U}(R_j)$ cannot find another relay

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`check_relay_availability`(R_j, η_m)

1: **if** R_j is not assigned to any user or $R_j = R_{k_m}$ **then**
2:   R_j is available.
3: **else**
4:   Set R_j as "marked"
5:   Run `find_another_relay`$(\mathcal{U}(R_j), R_j, \eta_m)$
6:   If $\mathcal{U}(R_j)$ finds another relay, then R_j is available, otherwise it is not
7: **end if**

As for computational complexity, [20] shows that generalized fractional programming to solve (7) has polynomial complexity, whereas [5] shows that the complexity of the user-relay association procedure is quadratic in K and N.

IV. NUMERICAL RESULTS AND DISCUSSION

A multi-user and relay-assisted network with $K = N = 10$ is considered. Fast fading is modeled as zero-mean, unit-variance circularly symmetric Gaussian complex random variables, while the channel-to-noise ratios of all the user-relay and relay-destination channels are $u = 10$ and $v = 100$, respectively. Also, $P_{k,n}^{(c)} = 0.1$ W for all k, $\mu_n = \lambda_k = 1$ for any k and n, $B = 1$ MHz, and $q_{max} = 10$ W for all n. Average results over 10^4 system realizations are reported.

Figs. 2 and 3 show the minimum energy efficiency in (7a) versus p_{max} (assumed the same for all users), for DF and AF

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[5]Recall that a 2×2 Hermitian matrix is non-negative semidefinite if its $(1,1)$-element is negative and if its determinant is non-negative.

[6]See [4] for further details about these functions.

$$\mathcal{H}_g(p_k, q_n) = \frac{1}{(p_k + q_n + 1)^2} \begin{pmatrix} -\dfrac{(q_n^2 + 2q_n + 2p_k q_n)}{(p_k + 1)^2} & 1 \\ 1 & -\dfrac{(p_k^2 + 2p_k + 2p_k q_n)}{(q_n + 1)^2} \end{pmatrix} \tag{8}$$

relaying protocols, respectively. The following algorithms are compared: (a) the proposed joint power and relay allocation; (b) optimal power allocation by solving (7), with a random relay assignment; (c) optimal relay assignment by solving (10), with full power transmission. It is seen that the joint allocation of the transmit powers and relay assignment significantly outperforms the optimization of only the transmit powers or the relay assignments.

Finally, Tab. I shows the average number of outer iterations for Algorithm 1 to converge, versus P_{max}. Convergence is declared when $|\min_k \mathrm{EE}_k^{(n)} - \min_k \mathrm{EE}_k^{(n-1)}|/\min_k \mathrm{EE}_k^{(n)} \leq 10^{-3}$, with $\min_k \mathrm{EE}_k^{(n)}$ denoting the minimum energy efficiency achieved in iteration n. The results indicate that convergence occurs in only a few iterations.
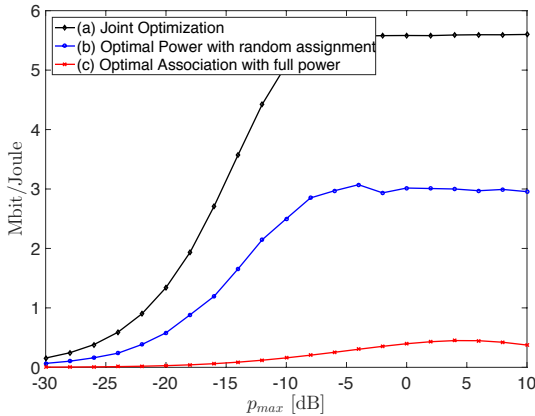

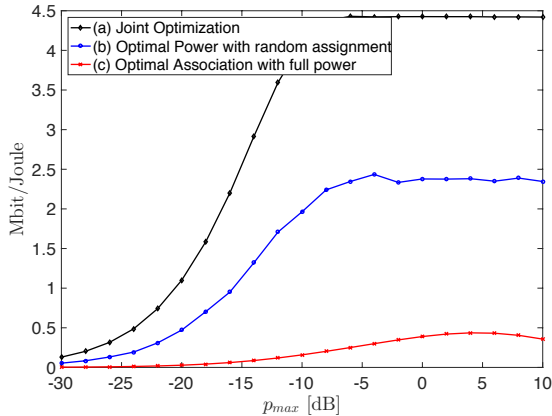Fig. 2: Minimum energy efficiency vs. p_{max} for DF relaying.

Fig. 3: Minimum energy efficiency vs. p_{max} for AF relaying.

P_{max} [dB]	-30	-20	-10	0	10
No. Iterations for AF	2.82	2.89	3.71	3.85	3.94
No. Iterations for DF	2	2.24	2.95	3.07	3.19

TABLE I: Algorithm 1. Average iterations number vs. P_{max}.

V. CONCLUSION

An algorithm for joint relay relay assignment and transmit power control has been developed to maximize the minimum among the users' energy efficiencies. The proposed approach merges alternating optimization and fractional programming theory, exhibiting a low complexity, while at the same time improving the energy efficiency compared to the optimization of only either the relay assignment or the transmit powers.

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