

Distributed Energy-Efficient UL Power Control in Massive MIMO with Hardware Impairments and Imperfect CSI

Alessio Zappone,^{*} Luca Sanguinetti,^{†‡} Giacomo Bacci,[§] Eduard Jorswieck,^{*} and Mérouane Debbah[¶]

^{*} Technische Universität Dresden, 5G Lab Germany, Dresden, Germany

[†] Large Systems and Networks Group (LANEAS), CentraleSupélec, Gif-sur-Yvette, France

[‡] Dipartimento di Ingegneria dell'Informazione, University of Pisa, Pisa, Italy

[§] Mediterranean Broadband Infrastructure (MBI) srl, Pisa, Italy

[¶] Mathematical and Algorithmic Sciences Lab, Huawei France, Paris, France

Abstract—This work proposes a distributed power allocation scheme to maximize energy efficiency in the uplink of multi-cell massive MIMO systems with hardware impairments at the user equipments (UEs) and imperfect channel state information at the base stations (BSs). Each UE in the network is modeled as a rational agent that engages in a generalized non-cooperative game and allocates its available transmit power to maximize its individual utility (defined as the UE's throughput per Watt of transmit power) subject to target rates and power constraints. The existence and uniqueness of the generalized Nash equilibrium of the game are studied in the asymptotic regime where the number of BS antennas and UEs grow large with a non trivial ratio. A fully distributed algorithm based on best-response dynamics and relying on large-scale fading components is proposed. Sufficient conditions to guarantee convergence to the equilibrium point are given. Numerical results are used to evaluate the performance of the proposed solution and to validate the analysis in a system of finite size.

I. INTRODUCTION

To satisfy the unrelenting demand for high-speed ubiquitous communications foreseen for the forthcoming decade, broadband wireless systems heavily rely on state-of-the-art digital signal processing techniques to transport broadband data through a very challenging wireless channel [1]. Among the most promising technologies, massive multiple-input multiple-output (MIMO) architectures represent a viable solution to meet the ambitious goals of future 5G networks while keeping the complexity of signal processing at the user equipments (UEs) at a tolerable level [2], [3].

Another peculiar feature of future cellular networks will be the massive amount of connected devices, which is growing at an exponential rate [4]. In this context, it is no longer possible to perform joint system designs by coordinating the transmissions of all nodes in the network. Rather, distributed algorithms to be implemented in networks with self-configuring nodes are becoming a necessity. Using this approach, the

nodes in a network are modeled as autonomous decision-makers, which compete with each other over the network resources. A well-established mathematical tool to analyze the interactions among competing entities is game theory [5], which has been largely employed for the development of distributed resource allocation schemes in wireless networks [6], [7]. Among the recent contributions in this field, we mention [8] wherein the authors study the Nash equilibrium problem for a group of players aiming at maximizing their own energy efficiency (EE) while satisfying power constraints in single and multi-carrier systems (similarly to what was done in [9] for rate maximization). A quasi-variational inequality approach is taken in [10] to develop power control algorithms in networks with heterogenous users. In [11], a similar problem is considered in the context of relay-assisted systems, whereas single-user multiple-input multiple-output (MIMO) networks are considered in [12]. It should be observed that most of existing works dealing with the maximization of EE do not account for rate requirements. This may translate into fairly low UE rates at the equilibrium. Incorporating target rates changes the setting drastically since any UE's admissible power allocation policy depends crucially on the policies of all other UEs. First results in this context are provided in [13] wherein Nash equilibria are found to be the fixed points of a water-filling best response operator whose water level depends on rate constraints and circuit power.

The aim of this work is to develop a distributed power allocation scheme for EE maximization in the uplink (UL) of a massive MIMO network. To this end, the UEs in the network are modeled as rational, self-organizing agents that engage in a non-cooperative game wherein each one aims at maximizing its individual EE while targeting its own power and rate constraints. This is done under the realistic assumptions that UEs are subject to transceiver hardware impairments [3] and that imperfect CSI is available at the BSs. Differently from [14] in which the number of BS antennas N can be arbitrarily large while the number of UEs K is kept fixed, the analysis is conducted for N and K that grow large with a given ratio K/N . The existence and uniqueness of the Nash equilibrium points of the asymptotic game are studied. A fully distributed algorithm based on best-response dynamics and operating at

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the rate of the large-scale fading evolution is proposed to reach equilibrium.

II. SYSTEM MODEL AND PROBLEM FORMULATION

This section introduces the system model and formulates the EE problem.

A. System Model

Consider the UL of a multi-cell multi-user MIMO system composed of L BSs with N antennas each and K single-antenna UEs in each cell. A double index notation is used to refer to each UE. In particular, the double-index kj refers to “user k in cell j ”. Since the signal from UE kj is received at its intended BS j as well as at all other BSs $\ell \neq j$, we call $\mathbf{h}_{k\ell} \in \mathbb{C}^N$ the channel vector from user kj to BS ℓ and assume that it is modeled as

$$\mathbf{h}_{k\ell} = \sqrt{d_{k\ell}} \mathbf{w}_{k\ell} \quad (1)$$

wherein $\mathbf{w}_{k\ell} \in \mathbb{C}^N$ is the small-scale fading channel modeled as a Gaussian vector with zero mean and covariance matrix $1/N \mathbf{I}_N$, and $d_{k\ell}$ accounts for the corresponding large-scale channel fading or path-loss (from UE kj to BS ℓ).

We assume that the transmission at UEs is affected by hardware impairments due to the use of non-ideal hardware. Following [3], this can be modeled as a reduction of useful signal power by a term $1 - \epsilon^2$ and by an additional Gaussian noise term $\eta_{kj} \sim \mathcal{CN}(0, \epsilon^2)$. In this setting, the transmitted signal x_{kj} of user kj can be written as

$$x_{kj} = \sqrt{p_{kj}(1 - \epsilon^2)} s_{kj} + \sqrt{p_{kj}} \eta_{kj} \quad (2)$$

where $s_{kj} \in \mathbb{C}$ is the useful signal of UE kj such that $\mathbb{E}\{|s_{kj}|^2\} = 1$ and p_{kj} accounts for the transmit power. As a result, the signal $\mathbf{y}_j \in \mathbb{C}^N$ received at BS j takes the form

$$\begin{aligned} \mathbf{y}_j &= \sqrt{p_{kj}(1 - \epsilon^2)} \mathbf{h}_{kjj} s_{kj} + \sqrt{p_{kj}} \mathbf{h}_{kjj} \eta_{kj} + \mathbf{n}_j + \\ &+ \sum_{(i,m) \neq (k,j)} \mathbf{h}_{imj} \left(\sqrt{p_{im}(1 - \epsilon^2)} s_{im} + \eta_{im} \right) \end{aligned} \quad (3)$$

with $\mathbf{n}_j \sim \mathcal{CN}(0, \sigma^2 \mathbf{I}_N)$ being the thermal noise. We assume that data recovery is accomplished by means of maximum-ratio-combining (MRC) under the realistic assumption that channel vectors are not perfectly estimated. We denote by $\hat{\mathbf{h}}_{kjj}$ the estimate of \mathbf{h}_{kjj} and assume that [15]

$$\hat{\mathbf{h}}_{kjj} = \sqrt{d_{kjj}} \left(\sqrt{1 - \tau_{kjj}^2} \mathbf{w}_{kjj} + \tau_{kjj} \mathbf{z}_{kjj} \right) \quad (4)$$

$$= \sqrt{1 - \tau_{kjj}^2} \mathbf{h}_{kjj} + \tau_{kjj} \mathbf{q}_{kjj} \quad (5)$$

where $\mathbf{z}_{kjj} \sim \mathcal{CN}(0, \mathbf{I}_N/N)$ accounts for the independent channel estimation errors, while the parameter τ_{kjj} reflects the accuracy or quality of the channel estimate, i.e., $\tau_{kjj} = 0$ for perfect CSI and $\tau_{kjj} = 1$ for a channel estimate completely uncorrelated to the genuine channel. Under the above assumptions, a lower bound of the SINR of UE kj at its serving BS

j can be computed as [16]:

$$\text{SINR}_{kj} = \frac{p_{kj}(1 - \epsilon^2) \left| \mathbb{E} \left[\hat{\mathbf{h}}_{kjj}^H \mathbf{h}_{kjj} \right] \right|^2}{\sigma^2 \mathbb{E} \left[\|\hat{\mathbf{h}}_{kjj}\|^2 \right] + p_{kj} A_{kjj} + \sum_{(i,m) \neq (k,j)} \mathbb{E} \left[\left| \hat{\mathbf{h}}_{kjj}^H \mathbf{h}_{imj} \right|^2 \right] p_{im}} \quad (6)$$

wherein A_{kjj} is given by

$$A_{kjj} = \epsilon^2 \mathbb{E} \left[\left| \hat{\mathbf{h}}_{kjj}^H \mathbf{h}_{kjj} \right|^2 \right] + (1 - \epsilon^2) \text{var} \left[\hat{\mathbf{h}}_{kjj}^H \mathbf{h}_{kjj} \right]. \quad (7)$$

B. Problem Formulation

The EE (measured in bit/Joule) of user kj is defined as the ratio of the achievable rate and the total consumed power [17], [18]

$$\text{EE}_{kj} \triangleq \frac{B \log_2(1 + \text{SINR}_{kj})}{p_{kj}^{(c)} + p_{kj}} \quad (8)$$

where B is the bandwidth and $p_{kj}^{(c)}$ is the circuit power dissipated to operate the kj -th transceiver (see for example [19] for more details on the power consumption model). To limit the transmit power, we assume that $p_{kj} - \bar{p}_{kj} \leq 0$ where \bar{p}_{kj} denotes user kj 's maximum power. Moreover, since an unconstrained EE maximization might lead to low spectral efficiencies per UE, we assume that minimum achievable rates need to be satisfied:

$$\log_2(1 + \text{SINR}_{kj}) - \underline{\theta}_{kj} \geq 0 \quad (9)$$

where $\underline{\theta}_{kj}$ is the target rate of user kj in [bit/s/Hz/UE]. Within the above setting, the goal of this work is to develop a decentralized power control algorithm for EE maximization. Mathematically, we aim at solving [8], [13]:

$$\arg \max_{p_{kj} \in \mathcal{A}_{kj}} \text{EE}_{kj}(p_{kj}, \mathbf{p}_{-kj}) \quad \forall kj \quad (10)$$

where \mathbf{p}_{-kj} is the interference vector containing all transmit powers except user kj 's, and \mathcal{A}_{kj} is the feasible set of p_{kj} given by:

$$\mathcal{A}_{kj} \triangleq \{p_{kj} \in \mathbb{R}_+ : p_{kj} \leq \bar{p}_{kj}, \log_2(1 + \text{SINR}_{kj}) \geq \underline{\theta}_{kj}\}. \quad (11)$$

The problem in (10) can be restated as a noncooperative game with complete information¹ defined as $\mathcal{G}' = [\mathcal{K}, \{\mathcal{A}_{kj}\}, \{\text{EE}_{kj}\}]$ in which: $\mathcal{K} = [1, \dots, KL]$ is the player set; \mathcal{A}_{kj} denotes the strategy set of player kj ; and EE_{kj} is player kl 's payoff function defined as in (8). Observe that not only EE_{kj} but also \mathcal{A}_{kj} depends on the other users' transmit powers \mathbf{p}_{-kj} through SINR_{kj} in (6). This makes \mathcal{G}' fall within the class of *generalized* non-cooperative games whose most widely used solution concept is known as generalized Nash equilibrium (GNE) [20].

To reduce the implementation complexity², we exploit the large-scale nature of the network and conduct the analysis in the asymptotic regime in which $N, K \rightarrow \infty$ with

¹In the context of game theory a non-cooperative game is said to have complete information if the players know the other players utility functions and strategy sets.

²The result to follow apply also when the SINR (6) is used.

$\lim_{N,K \rightarrow \infty} K/N \in (0, 1)$. This allows us to compute the so-called deterministic equivalents of $\{\text{SINR}_{kj}\}$ [21], as shown in the following lemma:

Lemma 1: If $N, K \rightarrow \infty$ with $\lim_{N,K \rightarrow \infty} K/N \in (0, 1)$, then $\max_{kj} |\text{SINR}_{kj} - \gamma_{kj}| \rightarrow 0$ almost surely with

$$\gamma_{kj} = \frac{\alpha_{kj} p_{kj}}{\sigma^2 + \phi_{kj} p_{kj} + \sum_{(i,m) \neq (k,j)} \omega_{kj,im} p_{im}} \quad (12)$$

where $\alpha_{kj} = (1 - \epsilon^2)(1 - \tau_{kjj}^2) d_{kjj}$, $\phi_{kj} = \epsilon^2(1 - \tau_{kjj}^2) d_{kjj}$, and $\omega_{im,kj} = \frac{1}{N} d_{imj}$.

Proof: The proof easily follows using standard results in random matrix theory [21], [22]. If $N, K \rightarrow \infty$ with $\lim_{N,K \rightarrow \infty} K/N \in (0, 1)$, then $\|\hat{\mathbf{h}}_{kjj}\|^2 - d_{kjj} \rightarrow 0$,

$$\hat{\mathbf{h}}_{kjj}^H \mathbf{h}_{kjj} - \sqrt{1 - \tau_{kjj}^2} d_{kjj} \rightarrow 0 \quad (13)$$

$$|\hat{\mathbf{h}}_{kjj}^H \mathbf{h}_{imj}|^2 - \frac{d_{kjj} d_{imj}}{N} \rightarrow 0 \quad (14)$$

almost surely. Also, A_{kjj} is such that $A_{kjj} - \epsilon^2 d_{kjj}^2 (1 - \tau_{kjj}^2) \rightarrow 0$ almost surely. Putting the above results together (12) easily follows. ■

Observe that the denominator of the SINR expression in (12) depends on the p_{kj} . This makes the analysis more involved compared to standard SINR expressions in which this does not occur.

III. GAME-THEORETIC FORMULATION AND ANALYSIS FOR LARGE SCALE NETWORKS

Using the above results, in the sequel we study the asymptotic game \mathcal{G} defined as $\mathcal{G} = [\mathcal{K}, \{\mathcal{P}_{kj}\}, \{\eta_{kj}\}]$ with

$$\eta_{kj} \triangleq \frac{B \log_2(1 + \gamma_{kj})}{p_{kj}^{(c)} + p_{kj}} \quad (15)$$

and $\mathcal{P}_{kj} \triangleq \{p_{kj} \in \mathbb{R}_+ : p_{kj} \leq \bar{p}_{kj}, \log_2(1 + \gamma_{kj}) \geq \underline{\theta}_{kj}\}$.

For later convenience, we define $\underline{\gamma}_{kj} = 2^{\underline{\theta}_{kj}} - 1$ the minimum SINR required to meet user kj 's rate constraint and $\bar{\gamma}_{kj} = \alpha_{kj}/\phi_{kj}$ the maximum SINR that user kj can obtain in the ideal case of zero multi-user interference and unlimited transmit power. Moreover, we call the equivalent channel gain of user kj :

$$\mu_{kj} \triangleq \frac{\alpha_{kj}}{\sigma^2 + \sum_{(m,i) \neq (j,k)} \omega_{im,kj} p_{im}} \quad (16)$$

A. Feasibility

We begin by looking at the feasibility conditions of the game. The following result is found:

Lemma 2: If

$$\bar{p}_{kj} \geq \underline{\gamma}_{kj} \frac{\sigma_{kj}^2 + \sum_{(i,m) \neq (k,j)} \omega_{im,kj} \bar{p}_{im}}{\alpha_{kj} - \phi_{kj} \underline{\gamma}_{kj}} \quad \forall kj \quad (17)$$

then $\mathcal{B}_{kj}(\mathbf{p}_{-kj})$ takes the form

$$\mathcal{B}_{kj}(\mathbf{p}_{-kj}) = \min \left\{ \bar{p}_{kj}, \max \left\{ p_{kj}^*, \underline{p}_{kj} \right\} \right\} \quad (18)$$

wherein

$$\underline{p}_{kj} \triangleq \frac{\underline{\gamma}_{kj}}{\mu_{kj}} \left(1 - \frac{\underline{\gamma}_{kj}}{\bar{\gamma}_{kj}} \right)^{-1} \quad (19)$$

and

$$p_{kj}^* \triangleq \arg \max_{p_{kj} \in \mathbb{R}_+} \eta_{kj}(p_{kj}, \mathbf{p}_{-kj}). \quad (20)$$

Proof: The first part of the thesis easily follows from rewriting the rate constraints $\gamma_{kj} \geq \underline{\gamma}_{kj}$ as

$$p_{kj} \geq \underline{\gamma}_{kj} \frac{\sigma^2 + \sum_{(i,m) \neq (k,j)} \omega_{im,kj} p_{im}}{\alpha_{kj} - \phi_{kj} \underline{\gamma}_{kj}}. \quad (21)$$

Since $p_{kj} \leq \bar{p}_{kj}$ for all $kj \in \mathcal{K}$, then

$$\begin{aligned} \underline{\gamma}_{kj} \frac{\sigma^2 + \sum_{(i,m) \neq (k,j)} \omega_{im,kj} \bar{p}_{im}}{\alpha_{kj} - \phi_{kj} \underline{\gamma}_{kj}} &\geq \\ \underline{\gamma}_{kj} \frac{\sigma^2 + \sum_{(i,m) \neq (k,j)} \omega_{im,kj} p_{im}}{\alpha_{kj} - \phi_{kj} \underline{\gamma}_{kj}} &. \end{aligned} \quad (22)$$

Hence, if $\forall kj \in \mathcal{K}$ (17) holds, then there always exists a power $p_{kj} \in [0, \bar{p}_{kj}]$ such that $\gamma_{kj} \geq \underline{\gamma}_{kj}$ is fulfilled. The last part of the proof follows by leveraging [11], where it is shown that for any given \mathbf{p}_{-kj} , η_{kj} is unimodal and thus admits a unique maximizer $p_{kj} \in \mathbb{R}_+$. Accounting for the power and rate constraints and imposing (17) eventually yields (18). ■

B. Analysis of the Equilibria

The existence and uniqueness of the GNE points of \mathcal{G} are now studied under the assumption that (17) holds.

Proposition 1: The game \mathcal{G} admits a nonempty set of GNE points.

Proof: Observe that the existence of a GNE is guaranteed under the following assumptions [23]:

- 1) The players' feasible action sets $\mathcal{P}_{kj}(\mathbf{p}_{-kj})$ are nonempty, closed, convex, and contained in some compact set \mathcal{C}_k for all $\mathbf{p}_{-kj} \in \mathcal{P}_{-kj} \equiv \prod_{(i,m) \neq (k,j)} \mathcal{P}_{im}$.
- 2) The sets $\mathcal{P}_k(\mathbf{p}_{-kj})$ vary continuously with \mathbf{p}_{-kj} (in the sense that the graph of the set-valued correspondence $\mathbf{p}_{-kj} \mapsto \mathcal{P}_{kj}(\mathbf{p}_{-kj})$ is closed).
- 3) Each user's payoff function $\eta_k(p_{kj}, \mathbf{p}_{-kj})$ is quasi-concave in p_{kj} for all $\mathbf{p}_{-kj} \in \mathcal{P}_{-kj}$.

In our setting, if the sufficient condition (17) is satisfied, then the sets $\mathcal{P}_{kj}(\mathbf{p}_{-kj})$ are nonempty, convex³, closed and bounded for every \mathbf{p}_{-kj} . Moreover, each of them varies continuously with \mathbf{p}_{-kj} since the rate constraint $\log_2(1 + \gamma_{kj}) \geq \underline{\theta}_{kj}$ in $\mathcal{P}_{kj}(\mathbf{p}_{-kj})$ is itself continuous in \mathbf{p}_{-kj} . Finally, following [11] $\eta_{kj}(p_{kj}, \mathbf{p}_{-kj})$ is proved to be strictly pseudo-concave since it is given by the ratio between a strictly concave and a linear function. Since any strictly pseudo-concave function is also strictly quasi-concave, the third condition is fulfilled. ■

The following result shows that a unique generalized Nash equilibrium (GNE) exists, and that the best-response dynamics (BRD) always converges to such point.

Proposition 2: The game \mathcal{G} admits a unique GNE point, which can be obtained by starting from any feasible power vector and iteratively updating the transmit powers according to (18).

Proof: See [24]. ■

³Note that the constraint function $\log_2(1 + \gamma_{kj})$ is concave in p_{kj} .

Algorithm 1 Iterative algorithm to solve (10).

1: **initialize** $n = 0$ and $\forall kj p_{kj}[0] \in \mathbb{R}_+$ in the feasible set
2: **repeat**
3: **for** $k = 1$ to K and $\ell = 1$ to L **do**
4: **compute** $\mu_{kj}[n]$ using (28)
5: **use** $\mu_{kj}[n]$ to update $\underline{p}_{kj}[n]$ in (27)
6: **use** $\mu_{kj}[n]$ in (25) to run the Dinkelbach algorithm
7: **set** $\lambda_{kj}^*[n]$ equal to the Dinkelbach output and update the power as:

$$p_{kj}[n+1] = \min \left\{ \bar{p}_{kj}, \max \left\{ \pi_{kj}(\lambda_{kj}^*[n]), \underline{p}_{kj}[n] \right\} \right\}$$

8: **end for**
9: **update** $n = n + 1$
10: **until** convergence

C. Distributed implementation

The best response of a generic player k is characterized in the sequel to come up with an iterative algorithm that allows each player to reach the GNE in a distributed manner. Toward this end, let us first define

$$\nu_k(x) \triangleq \bar{\gamma}_{kj} \left[1 + \frac{x}{2B\mu_{kj}} (\bar{\gamma}_{kj} - g_{kj}(x)) \right]^+ \quad (23)$$

and $g_{kj}(x) \triangleq \sqrt{\bar{\gamma}_{kj}^2 + \frac{4B\mu_{kj}}{x} (1 + \bar{\gamma}_{kj})}$, with μ_{kj} as in (16).

Lemma 3: For any given \mathbf{p}_{-kj} (or, equivalently, μ_{kj}), the solution to (20) is found to be

$$p_{kj}^* = \pi_{kj}(\lambda_{kj}^*) \triangleq \frac{\nu_{kj}(\lambda_{kj}^*)}{\mu_{kj}} \left(1 - \frac{\nu_{kj}(\lambda_{kj}^*)}{\bar{\gamma}_{kj}} \right)^{-1} \quad (24)$$

where λ_{kj}^* is obtained through the Dinkelbach method as the solution of the following equation:

$$B \log_2(1 + \nu_{kj}(\lambda_{kj}^*)) - \lambda_{kj}^*(p_{c,kj} + \pi_{kj}(\lambda_{kj}^*)) = 0. \quad (25)$$

Proof: See [24]. \blacksquare

Denote by $p_{kj}[n]$ the transmit power of the kj -th player at the n -th iteration step. Using the results of Proposition 2 and Lemma 3, it follows that an iterative algorithm operating according to

$$p_{kj}[n+1] = \min \left\{ \bar{p}_{kj}, \max \left\{ \pi_{kj}(\lambda_{kj}^*[n]), \underline{p}_{kj}[n] \right\} \right\} \quad (26)$$

where $\underline{p}_{kj}[n]$ is computed as (using (19))

$$\underline{p}_{kj}[n] = \frac{\gamma_{kj}}{\mu_{kj}[n]} \left(1 - \frac{\gamma_{kj}}{\bar{\gamma}_{kj}} \right)^{-1} \quad (27)$$

converges to the unique GNE of \mathcal{G} , with $\mu_{kj}[n]$ being the equivalent channel gain in (16) at the n -th iteration step. The pseudo-code is reported in Algorithm 1.

A close inspection of (23) – (25) and (27) reveals that the computation of $p_{kj}[n+1]$ through (26) only requires knowledge of $\mu_{kj}[n]$. Although not available at the kj -th terminal, this information can be easily acquired taking into account that:

$$\mu_{kj}[n] = \frac{\gamma_{kj}[n]}{p_{kj}[n]} \left(1 - \frac{\gamma_{kj}[n]}{\bar{\gamma}_{kj}} \right)^{-1} \quad (28)$$

where $\gamma_{kj}[n]$ denotes the SINR of transmitter kj measured at its intended receiver at iteration n . Since $p_{kj}[n]$ and $\bar{\gamma}_{kj}$ are locally available at the transmitter, the computation of $\mu_{kj}[n]$ only requires knowledge of $\gamma_{kj}[n]$. The latter only depends on large-scale fading components $\{d_{kj\ell}\}$, which can be accurately estimated and easily exchanged between BSs as they change slowly with time (relative to the small-scale fading). Therefore, besides being guaranteed to converge to the unique GNE, Algorithm 1 can also be implemented in a fully decentralized fashion at a rate of the large-scale fading evolution.

IV. NUMERICAL RESULTS

In our simulations we consider a four-cell system, wherein each base station deploys $N = 64$ antennas and serves $K = 8$ users. Each cell is a square with edge 500 m, wherein the served users are randomly distributed, with a minimum distance of 50 m from the service base station. All users have the same maximum feasible power \bar{P} and hardware-dissipated power $p^{(c)} = 10$ dBm. The receive noise power is $\sigma^2 = FB\mathcal{N}_0$, with $F = 3$ dB, $B = 1$ MHz, and $\mathcal{N}_0 = -174$ dBm/Hz. All channels are generated according to Rayleigh fading model with path-loss model as in [25]. We set the channel estimation accuracy factor as $\tau = 0.3$ and the hardware impairment factor to $\epsilon = 0.1$. The minimum rate constraint $\underline{\theta}_k$ is set as a percentage R_k of the maximum rate that user k can achieve when $p_k \rightarrow \infty$, while the other users' powers are finite, namely:

$$\underline{\theta}_k = R_k \lim_{p_k \rightarrow \infty} \log_2(1 + \gamma_{kj}) = R_k \log_2(1 + \alpha_{kj}/\phi_{kj}) \quad (29)$$

For simplicity, we assume $R_1 = R_2 = \dots = R_K = R$.

As performance measure to assess the performance of the proposed schemes we consider the network global energy efficiency (GEE), defined as [18]

$$\text{GEE} = \frac{\sum_{k,j} B \log_2(1 + \gamma_{kj})}{\sum_{k,j} p_{kj}^{(c)} + p_{kj}}. \quad (30)$$

In Fig. 1 we compare the GEE achieved by the the following resource allocation schemes:

- Algorithm 1, with $R = 20\%$. In case one best-response is unfeasible, we relax the rate requirement to $R = 0\%$;
- Algorithm 1, with $R = 10\%$. If one best-response is unfeasible, we relax the rate requirement to $R = 0\%$;
- Algorithm 1, with $R = 0\%$.
- As a benchmark, we report the GEE obtained by a centralized approach designed to maximize the GEE [24]. In this case we consider $R = 0\%$.

For low values of \bar{P} , all schemes perform similarly, because when \bar{P} is low, the problems with rate requirement are likely to be unfeasible, and in this case schemes (a) and (b) are equivalent to (c). Besides, for low \bar{P} , we see that coordinating the interference among the different transmitters does not bring any significant advantage. Instead, for increasing \bar{P} , the centralized scheme (d) which jointly manages the multi-user interference outperforms the distributed schemes, which suffer from the so-called price-of-anarchy effect [7]. Also, enforcing

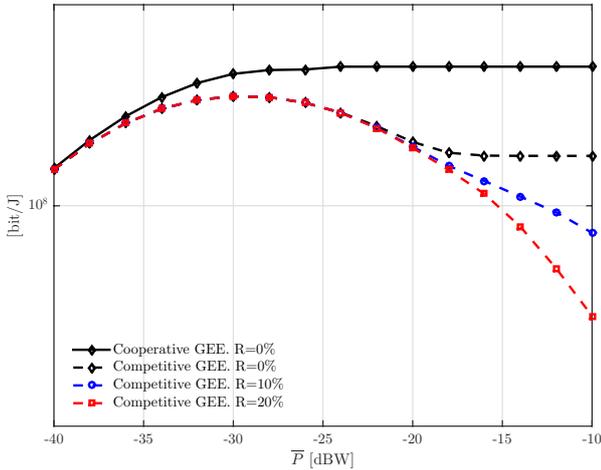


Figure 1. $K = 8; N = 64; L = 4; \epsilon = 10^{-1}; \tau = 0.3$. GEE versus \bar{P} for: (a) Algorithm 1 with $R = 20\%$; (b) Algorithm 1 with $R = 10\%$; (c) Algorithm 1 with $R = 0\%$; (d) GEE maximization with $R = 0\%$ [24].

Table I
 $K = 8; N = 64; L = 4; \epsilon = 10^{-1}; \tau = 0.3$. AVERAGE NUMBER OF REQUIRED ITERATIONS TO REACH CONVERGENCE VERSUS \bar{P} FOR: (A) ALGORITHM 1 WITH $R = 20\%$; (B) ALGORITHM 1 WITH $R = 10\%$; (C) ALGORITHM 1 WITH $R = 0\%$.

Rate requirement	$R = 0\%$	$R = 10\%$	$R = 20\%$
$\bar{P} = -40$ [dBW]	1	1	1
$\bar{P} = -32$ [dBW]	1.03	1.03	1.03
$\bar{P} = -24$ [dBW]	2.30	2.35	2.47
$\bar{P} = -16$ [dBW]	4.81	4.93	5.05
$\bar{P} = -8$ [dBW]	5.10	5.22	5.37

rate constraints results in a lower GEE, because the mobiles need to use the excess transmit power in order to meet their rate requirement.

Next, we analyze the computational complexity of Algorithm 1. Table I shows the average number of iterations required by Algorithm 1 to converge, for $R = 0\%$, $R = 10\%$, and $R = 20\%$. The rule $\|\mathbf{p}^{(n)} - \mathbf{p}^{(n-1)}\|^2 / \|\mathbf{p}^{(n)}\|^2 \leq 10^{-4}$ is used to declare convergence. It is seen that convergence occurs after a handful of iterations, which slightly increases for larger \bar{P} , since increasing \bar{P} results in a larger feasible set. This shows that the proposed non-cooperative approach lends itself to a simple implementation in practical systems. Moreover, for low \bar{P} , the number of iterations is not affected by R , because in this range the generalized game does not reach an equilibrium and the result of the regular non-cooperative game is used. The impact of R continues to be minimal also for higher \bar{P} .

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