

Slow Admission and Power Control for Small Cell Networks via Distributed Optimization

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Abstract—Although small cell networks are environmentally friendly and can potentially improve the coverage and capacity of cellular layers, it is imperative to control the interference in such networks before overlaying them in a macrocell network on a large-scale basis. In recent work, we developed the joint admission and power control algorithm for two-tier small cell networks in which the number of small cell users that can be admitted at their quality-of-service (QoS) constraints is maximized without violating the macrocell users' QoS constraints. The QoS metric adopted is outage probability. In this paper, we investigate the distributed implementation of the joint admission and power control problem where the small cells can determine jointly their admissibility and transmit powers autonomously.

I. INTRODUCTION

Energy consumption and electromagnetic pollution are fast becoming problems that future communication infrastructures need to alleviate. Towards this end, the design of green cellular networks has been considered. One such approach is to overlay a macrocell network with many small cells [1], [2]. With small cells, users can obtain better indoor reception and power savings due to the low transmit powers. In addition, small cells can offload much data traffic from the macrocell network via backhaul. This enhances the overall network coverage and capacity. However, one of the major impediments to the success of two-tier small cell networks is the presence of inter-tier and intra-tier interference.

A significant amount of research is focused on managing inter-tier and intra-tier interference [3]–[15]. In [3], inter-tier interference between the macrocell and small cell tiers can be avoided by using orthogonal spectrum allocation. Clearly, this method is inefficient given a sparse small cell deployment setting and a much higher area spectrum efficiency can be achieved by spectrum sharing [4]. On the other hand, for spectrum sharing in two-tier small cell networks, the inter-tier interference has to be properly controlled by using techniques such as access control [4]–[7], power control [8]–[10], multiple antennas [11], [12], or cognitive radio [13]–[15]. All these schemes [3]–[15] involve computational and signalling overhead. If the set of active small cells changes at the rate of Rayleigh fading, there will be very frequent updating and processing at the macrocell base station (MBS) and small cell access points (SAPs). Therefore, we proposed an interference management scheme with joint admission and power control that tracks at a much slower shadowing time-scale in [16].

In [16], we consider a two-tier small cell network, where small cell users (SUs) share the same spectrum with the macrocell user (MU). We assume that the MU has a higher spectrum access priority than the SUs. The proposed joint admission and power control problem aims to maximize the number of small cells admitted with their outage probability constraints satisfied and simultaneously, minimize their total transmit power, while guaranteeing the outage probability constraint of the MU. Different from conventional works [17], [18] which use instantaneous signal-to-interference-plus-noise ratio (SINR) constraints, we apply outage probability constraint for the users because this enables the admission and power updating to be performed at a much slower log-normal shadowing time-scale instead of the Rayleigh fading time-scale. As this formulation is NP-hard, convex relaxation is applied to obtain high quality approximate solutions that exhibit near optimal performance as shown in [16]. However, the centralized approach in [16] is difficult to implement in practice as the SAPs are randomly deployed by their subscribers.

In this paper, we investigate the decentralized implementation of the algorithm in [16] which is developed via dual decomposition. The distributed algorithm enables the small cells to self organize; the small cells can determine jointly (on their own) if they are admitted or rejected for communications and the respective transmit powers should they be admitted into the system. The simulation results show the effectiveness of our proposed distributed algorithm.

II. SYSTEM MODEL AND PROBLEM FORMULATION

As shown in Fig. 1, we consider an uplink two-tier network where a macrocell network is overlaid with N closed access small cells. The MBS and the SAPs are operating in a common frequency band with one MU¹ and N SUs. We assume that there is one SU in each small cell requesting to share the spectrum with the MU in order to communicate with its SAP. Therefore, the received SINR of the i th SU can be written as

$$\text{SINR}_i^s = \frac{G_{ii}^{ss} F_{ii}^{ss} P_i^s}{\sum_{l=1, l \neq i}^N G_{il}^{ss} F_{il}^{ss} P_l^s + G_{i0}^{sm} F_{i0}^{sm} P^m + N_o} \quad (1)$$

¹A single MU is considered for brevity of exposition. More QoS constraints can be added to include multiple MUs which does not alter the structure of the proposed problem.

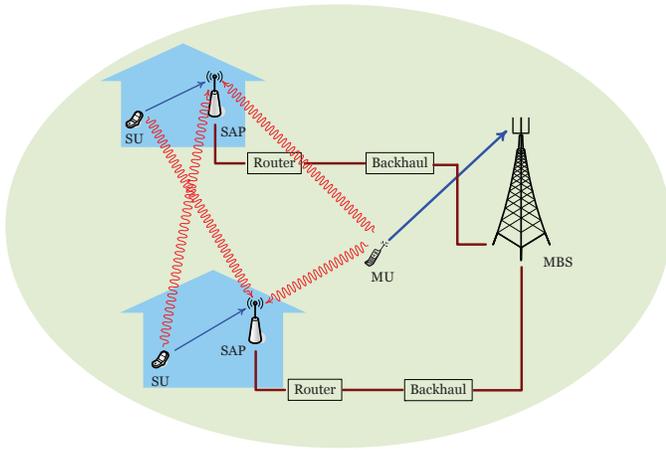


Fig. 1. Two-tier network where a macrocell network is overlaid with small cells. The straight arrows indicate the desired links while the wavy arrows indicates inter-tier and intra-tier interfering links.

where G_{il}^{ss} and F_{il}^{ss} denote the slow and fast fading gains from the l th SU to the i th SAP, respectively. In the following, we consider the slow fading gain to include the effect of propagation path loss and shadowing, and the fast fading gain is modeled as exponential power fading which corresponds to Rayleigh fading assumption. Similarly, G_{i0}^{sm} and F_{i0}^{sm} refers to the slow and fast fading gains from the MU (assigned index “0”) to the i th SAP, respectively. The transmit power of the l th SU is P_l^s , the transmit power of the MU is P^m , which is assumed to be fixed as the MU does not cooperate with the SUs, and the noise power is N_o . With the Rayleigh fading assumption on the fast fading gains, F_{ij}^{kl} are independent exponentially distributed random variables with unit mean. Thus, the outage constraint of the i th SU is given by $\Pr(\text{SINR}_i^s \leq \gamma_i^{s,\text{th}}) \leq \rho_i^{s,\text{th}}$ and derived as [10]

$$\frac{b_i^s N_o}{P_i^s} + \sum_{l=1, l \neq i}^N \ln\left(1 + \frac{b_i^s G_{il}^{ss} P_l^s}{P_i^s}\right) + \ln\left(1 + \frac{b_i^s G_{i0}^{sm} P^m}{P_i^s}\right) \leq -\ln(1 - \rho_i^{s,\text{th}}) \quad (2)$$

where $\gamma_i^{s,\text{th}}$ and $\rho_i^{s,\text{th}}$ denote the pre-specified SINR and outage probability thresholds of the i th SU, respectively, and for notational convenience, $b_i^s \triangleq \gamma_i^{s,\text{th}}/G_{ii}^{ss}$.

When SUs are operating, the received SINR of the MU is

$$\text{SINR}^m = \frac{G_{M0}^{mm} F_{M0}^{mm} P^m}{\sum_{i=1}^N G_{Mi}^{ms} F_{Mi}^{ms} P_i^s + N_o} \quad (3)$$

where G_{M0}^{mm} and F_{M0}^{mm} are the slow and fast fading gains from the MU to the MBS and G_{Mi}^{ms} and F_{Mi}^{ms} are the slow and fast fading gains from the i th SU to the MBS. The outage constraint of the MU is then given by $\Pr(\text{SINR}^m \leq \gamma^{m,\text{th}}) \leq \rho^{m,\text{th}}$ and derived as

$$\sum_{i=1}^N \ln(1 + b_{Mi}^{ms} P_i^s) \leq \ln \mu^m \quad (4)$$

where $\rho^{m,\text{th}}$ is the pre-specified outage probability threshold of the MU, $\mu^m = (1 - \bar{\rho}^m)/(1 - \rho^{m,\text{th}})$, $\bar{\rho}^m$ is the outage probability of the MU in the absence of SUs, and for notational convenience, $b_{Mi}^{ms} \triangleq (\gamma^{m,\text{th}} G_{Mi}^{ms})/(G_{M0}^{mm} P^m)$.

The objective of this work is to maximize the number of SUs that can be admitted with a guaranteed QoS while guaranteeing the QoS of the MU and simultaneously minimize the total transmission power of the SUs. In this paper, the QoS provided to the MU as well as the SUs is outage probability constraint. Following the approach in [17], we can provide a compact and elegant single-stage framework [16] which is given by

$$\min_{P_i^s, s_i} \quad \epsilon \sum_{i=1}^N P_i^s + (1 - \epsilon) \sum_{i=1}^N \frac{1}{s_i + 1} \quad (5a)$$

$$\text{s.t.} \quad 0 \leq P_i^s \leq P^{s,\text{max}}, \quad \forall i \quad (5b)$$

$$s_i \in [0, 1], \quad \forall i \quad (5c)$$

$$\frac{s_i b_i^s N_o}{P_i^s} + \sum_{l=1, l \neq i}^N \ln\left(1 + \frac{s_i b_i^s G_{il}^{ss} P_l^s}{P_i^s}\right) + \ln\left(1 + \frac{s_i b_i^s G_{i0}^{sm} P^m}{P_i^s}\right) \leq -\ln(1 - \rho_i^{s,\text{th}}), \quad \forall i \quad (5d)$$

$$\sum_{i=1}^N \ln(1 + b_{Mi}^{ms} P_i^s) \leq \ln \mu^m \quad (5e)$$

where the auxiliary scheduling variables are $s_i \in [0, 1]$, $P^{s,\text{max}}$ is the maximum transmit power of each SU, the value of s_i determines the admissibility of the i th SU and if the outage constraint of the i th SU is taken into consideration in the power control part of the joint admission and power control problem. If $s_i = 0$, the i th SU is rejected and (5d) reduces to the trivial inequality $\ln(1 - \rho_i^{s,\text{th}}) \leq 0$; if $s_i = 1$, the i th SU is scheduled for admission and (5d) becomes an active constraint. The cost function consists of the weighted sum of transmit powers of the SUs which is bounded and the admission cost which is discrete-valued. Intuitively, the weighing parameter $\epsilon < \epsilon^*$ has to be small enough in order to ensure that admission control is always prioritized before power control. The choice of ϵ can be understood by the following interpretation that dropping any user costs more than can possibly be saved by total power minimization [17].

Remark 1: By choosing $\epsilon < \epsilon^* = 1/(2N P^{s,\text{max}} + 1)$, the solution of (5) is equivalent to firstly, finding the optimal set $\tilde{\mathcal{S}}$ that contains the largest number of SUs that can be admitted subject to transmit power constraints (5b), outage probability constraints of the SUs (2), and outage probability constraint of the MU (4) and lastly, minimizing the total transmit power of the SUs in $\tilde{\mathcal{S}}$ subject to the same constraints (5b), (2), and (4).

Proof: The proof is omitted for brevity. \square

Different from [17] which constrains the instantaneous SINRs of the SUs, the proposed formulation constrains the outage probabilities of the MU and SUs. Consequently, the objective function, weighing parameter ϵ , and the effect that the scheduling variables s_i exert on the constraints in (5) are entirely different from those in [17].

III. CONVEX RELAXATION

The single-stage reformulation in (5) is non-convex due to the binary constraints (5c) and the term $1/(s_i + 1)$ in the

objective function being neither a posynomial nor a monomial. Therefore, we apply the technique of convex relaxation. First, we relax the binary constraints to allow s_i to take on any real value within the interval $[0, 1]$. Next, we approximate $f(s_i) = 1/(s_i + 1)$ with a monomial, i.e., $cs_i^\alpha = 0.5s_i^{-\frac{1}{2}}$ (details are skipped for brevity) such that the entire optimization problem can be cast as a geometric programming (GP) problem. Finally, the new convex single-stage formulation is obtained as follows:

$$\begin{aligned} \min_{\tilde{P}_i^s, \tilde{s}_i} & \epsilon \sum_{i=1}^N P_i^s + c(1 - \epsilon) \sum_{i=1}^N s_i^{-\alpha} & (6) \\ \text{s.t.} & 0 \leq s_i \leq 1, & \forall i \\ & (5b), (5d), (5e) \end{aligned}$$

which is clearly a GP and it can be solved globally and efficiently. After (6) is solved, if all $s_i = 1$, it means that all the MU and SUs can be served while satisfying their outage constraints. Otherwise, the removal of SUs is triggered in order to admit the maximum number of SUs with their outage constraints and that of the MU met. In this paper, the iterative removal algorithm is used to remove the SU with the minimal $s_i|_{s_i \neq 1}$ (one by one) at each iteration of (6). The algorithm terminates when $s_i = 1$ for all the remaining SUs.

IV. DISTRIBUTED OPTIMIZATION

Although (6) can be efficiently solved in polynomial time, it still requires a centralized network controller to solve the optimization problem. Due to the random deployment of the small cells, distributed algorithms are especially favoured so that the small cells can determine their own admissibility and transmit powers (if they are admitted for communications). To this end, the dual decomposition approach is applied [19]. As (5d) is coupled with the transmit powers of other SUs, an auxiliary variable $z_{il} = G_{il}^{ss} P_l^s$ and an additional equality (7d) are introduced. Let $\tilde{s}_i = \ln s_i$, $\tilde{P}_i^s = \ln P_i^s$, and $\tilde{z}_{il} = \ln z_{il}$, then we can rewrite (6) as follows:

$$\min_{\tilde{P}_i^s, \tilde{s}_i} \epsilon \sum_{i=1}^N \exp(\tilde{P}_i^s) + c(1 - \epsilon) \sum_{i=1}^N (\exp(\tilde{s}_i))^{-\alpha} \quad (7a)$$

$$\text{s.t.} \quad \tilde{P}_i^s \leq \ln P^{\text{s,max}}, \quad \forall i \quad (7b)$$

$$\tilde{s}_i \leq 0, \quad \forall i \quad (7c)$$

$$\tilde{z}_{il} - \ln G_{il}^{ss} - \tilde{P}_l^s = 0, \quad \forall i \neq l \quad (7d)$$

$$\begin{aligned} & \frac{\exp(\tilde{s}_i) b_i^s N_o}{\exp(\tilde{P}_i^s)} + \sum_{l=1, l \neq i}^N \ln \left(1 + \frac{\exp(\tilde{s}_i) b_i^s \exp(\tilde{z}_{il})}{\exp(\tilde{P}_i^s)} \right) \\ & + \ln \left(1 + \frac{\exp(\tilde{s}_i) b_i^s G_{i0}^{\text{sm}} P^{\text{m}}}{\exp(\tilde{P}_i^s)} \right) \leq -\ln(1 - \rho_i^{\text{s,th}}), \forall i \quad (7e) \end{aligned}$$

$$\sum_{i=1}^N \ln(1 + b_{M_i}^{\text{ms}} \exp(\tilde{P}_i^s)) \leq \ln \mu^{\text{m}}. \quad (7f)$$

Let γ_{il} be the consistency prices corresponding to (7d), ζ_i and λ be the dual variables for (7e) and (7f), respectively so the

partial Lagrangian for (7) can be written as

$$\begin{aligned} L(\tilde{P}_i^s, \tilde{s}_i, \tilde{z}_{il}) = & \epsilon \sum_{i=1}^N \exp(\tilde{P}_i^s) + c(1 - \epsilon) \sum_{i=1}^N (\exp(\tilde{s}_i))^{-\alpha} \\ & + \sum_{i=1}^N \zeta_i \left[\frac{\exp(\tilde{s}_i) b_i^s N_o}{\exp(\tilde{P}_i^s)} \right. \\ & + \sum_{l=1, l \neq i}^N \ln \left(1 + \frac{\exp(\tilde{s}_i) b_i^s \exp(\tilde{z}_{il})}{\exp(\tilde{P}_i^s)} \right) \\ & \left. + \ln \left(1 + \frac{\exp(\tilde{s}_i) b_i^s G_{i0}^{\text{sm}} P^{\text{m}}}{\exp(\tilde{P}_i^s)} \right) + \ln(1 - \rho_i^{\text{s,th}}) \right] \\ & + \lambda \left[\sum_{i=1}^N \ln(1 + b_{M_i}^{\text{ms}} \exp(\tilde{P}_i^s)) - \ln \mu^{\text{m}} \right] \\ & + \sum_{i=1}^N \sum_{l=1, l \neq i}^N \gamma_{il} [\tilde{z}_{il} - \ln G_{il}^{\text{ss}} - \tilde{P}_l^s] \quad (8) \end{aligned}$$

where for each SU, $i = 1, \dots, N$, we have

$$\begin{aligned} L_i(\tilde{P}_i^s, \tilde{s}_i, \tilde{z}_{il}) = & \epsilon \exp(\tilde{P}_i^s) + c(1 - \epsilon) (\exp(\tilde{s}_i))^{-\alpha} \\ & + \zeta_i \left[\frac{\exp(\tilde{s}_i) b_i^s N_o}{\exp(\tilde{P}_i^s)} \right. \\ & + \sum_{l=1, l \neq i}^N \ln \left(1 + \frac{\exp(\tilde{s}_i) b_i^s \exp(\tilde{z}_{il})}{\exp(\tilde{P}_i^s)} \right) \\ & \left. + \ln \left(1 + \frac{\exp(\tilde{s}_i) b_i^s G_{i0}^{\text{sm}} P^{\text{m}}}{\exp(\tilde{P}_i^s)} \right) \right] \\ & + \lambda \ln(1 + b_{M_i}^{\text{ms}} \exp(\tilde{P}_i^s)) \\ & + \sum_{l=1, l \neq i}^N \gamma_{il} \tilde{z}_{il} - \left(\sum_{l=1, l \neq i}^N \gamma_{li} \right) \tilde{P}_i^s. \quad (9) \end{aligned}$$

The consistency prices γ_{il} and Lagrange multipliers ζ_i and λ can be updated according to the following equations:

$$\gamma_{il}(t+1) = \gamma_{il}(t) + \delta_\gamma(t) [\tilde{z}_{il} - \ln G_{il}^{\text{ss}} - \tilde{P}_l^s], \quad (10)$$

$$\begin{aligned} \zeta_i(t+1) = & \zeta_i(t) + \delta_\zeta(t) \left[\frac{\exp(\tilde{s}_i) b_i^s N_o}{\exp(\tilde{P}_i^s)} \right. \\ & + \sum_{l=1, l \neq i}^N \ln \left(1 + \frac{\exp(\tilde{s}_i) b_i^s \exp(\tilde{z}_{il})}{\exp(\tilde{P}_i^s)} \right) \\ & \left. + \ln \left(1 + \frac{\exp(\tilde{s}_i) b_i^s G_{i0}^{\text{sm}} P^{\text{m}}}{\exp(\tilde{P}_i^s)} \right) + \ln(1 - \rho_i^{\text{s,th}}) \right], \quad (11) \end{aligned}$$

$$\lambda(t+1) = \lambda(t) + \delta_\lambda(t) \left[\sum_{i=1}^N \ln(1 + b_{M_i}^{\text{ms}} \exp(\tilde{P}_i^s)) - \ln \mu^{\text{m}} \right] \quad (12)$$

where t is the iteration index and δ_γ , δ_ζ , and δ_λ are step sizes, respectively.

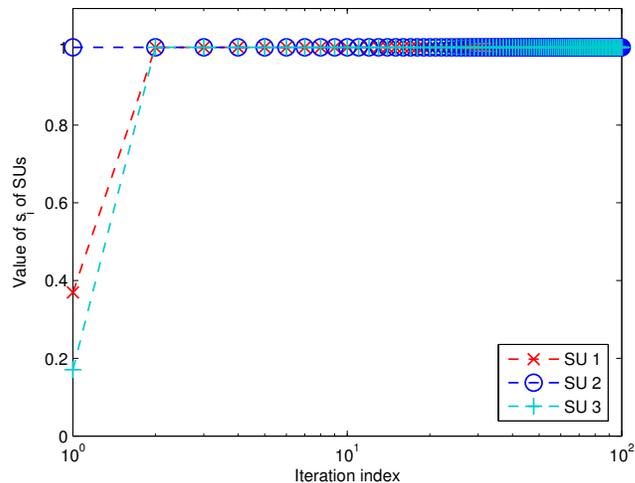
Incorporating all the above equations, we summarize our proposed distributed joint admission and power control algorithm as follows:

- 1) At $t = 0$, initialize $\gamma_{il} = 0$ for $\forall i, l$, $\lambda > 0$, $\zeta_i > 0$, and $\tilde{z}_{il} = 0$. First, each SAP i estimates the interference from the MU, i.e., $G_{i0}^{sm} P^m$, and channel gains from other SUs, i.e., G_{il}^{ss} . Each SAP i can get $\gamma^{m,th}$, G_{M0}^{mm} , P^m , and μ^m from the MBS using the backhaul. Each SU i can obtain G_{Mi}^{ms} via estimating G_{iM}^{sm} when the MBS is broadcasting or transmitting, after which it informs its SAP.
- 2) At $t \geq 1$, each SAP i solves (9) subject to its transmit power constraint (7b) for \tilde{P}_i^s and \tilde{s}_i using interior point method.
- 3) Each SAP i checks if \tilde{P}_i^s and \tilde{s}_i have converged. If so, go to Step (5). Otherwise, the SAP i broadcasts its \tilde{P}_i^s or passes this information to other SAPs over a low-rate control channel.
- 4) The SAPs can appoint one of themselves as an acting SAP controller to pass their G_{Mi}^{ms} to. The acting SAP controller updates λ and informs the other SAPs of λ via the low-rate control channel. Each SAP i updates \tilde{z}_{il} , γ_{il} , and ζ_i . Go to Step (2).
- 5) After \tilde{P}_i^s and \tilde{s}_i have converged, each SAP can calculate P_i^s and s_i . If $s_i = 1$, the SU i is admitted to share the spectrum with the MU and the SU i then transmits at P_i^s to communicate with the respective SAP. If $s_i = 0$, the SU i is rejected and it does not transmit at all.

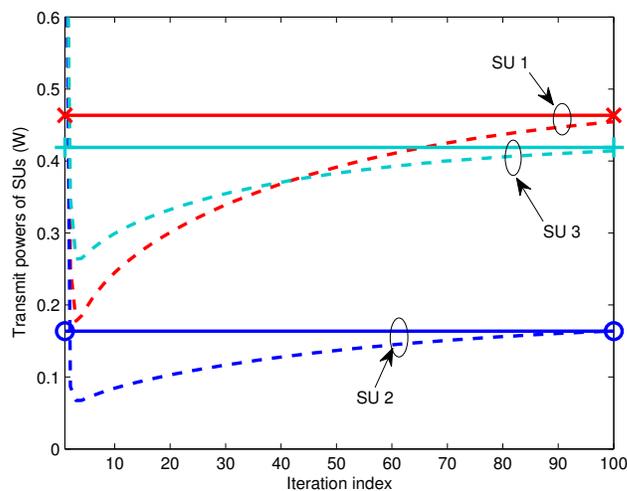
V. SIMULATION RESULTS

The performance of our proposed distributed joint admission and power control algorithm is investigated for a code division multiple access system. The MBS is located in the centre of a square area of length 2000m. The small cells are randomly located in the same area excluding a square area of length 100m centred at the MBS. The SAP is located at the centre of each small cell (square area) and the SU is randomly located at either one of the four corners of the cell at a distance of 20m. The small cells are separated from each other by at least 1m. The MU is randomly located outside the small cells by at least 1m. The noise power at the MBS and SAPs is $N_0 = 10^{-10}$ W. The transmit power and SINR threshold of the MU are $P^m = 4$ W and $\gamma^{m,th} = 0$ dB, respectively. The total number of requesting SUs is 3. The SINR thresholds of the SUs is 25dB. The maximum transmit power of the SUs is $P^{s,max} = 1$ W. The processing gains of the MBS and SAPs are $PG^m = 10$ and $PG_i^s = 1$, respectively. The MU and SUs have an outage probability threshold $\rho^{m,th} = 10\%$ and $\rho_i^{s,th} = 10\%$. The slow fading gain between transmitter j and receiver i is modeled as $G_{ij} = K_0 \times 10^{\beta_{ij}/10} \times d_{ij}^{-\eta}$ where d_{ij} is the distance between them, $K_0 = 10^3$ is a factor to include the effects of antenna gain and carrier frequency, β_{ij} is a Gaussian random variable with zero mean and standard deviation of 4dB to account for log-normal shadowing effects, and the path loss exponent is $\eta = 3$ except that $\eta = 4$ for G_{ii}^{ss} .

We study the performance of the proposed distributed joint admission and power control algorithm by comparing with its centralized counterpart. In the first example in Fig. 2, the centralized algorithm admits all the requesting SUs while in the second example in Fig. 3, the centralized algorithm admits



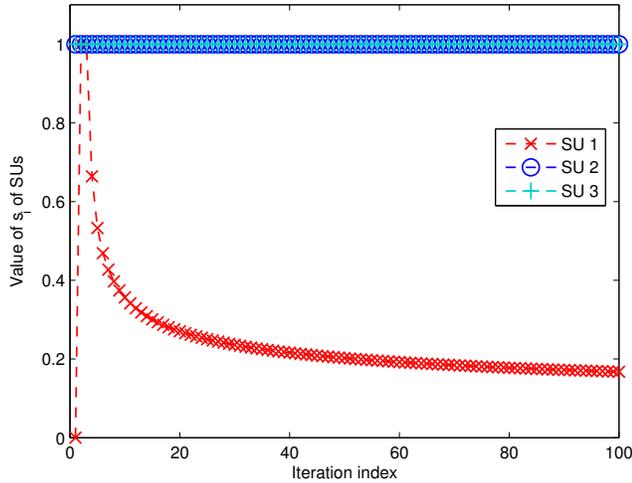
(a)



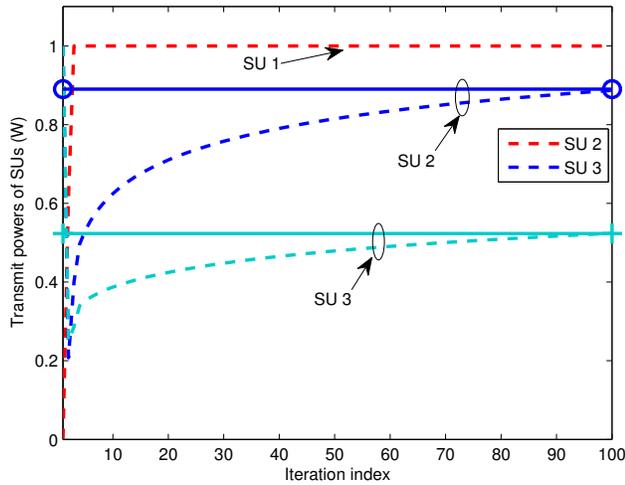
(b)

Fig. 2. (a) Value of s_i of SUs obtained by the proposed distributed algorithm versus iteration index. All SUs are admitted. Note that the x-axis is on a logarithm scale. (b) Transmit powers of SUs obtained by the proposed distributed algorithm versus iteration index. The solid lines indicate the transmit powers of the respective SUs as obtained by the centralized algorithm.

only 2 SUs which are SU 2 and SU 3. Fig. 2(a) and Fig. 3(a) show that the distributed algorithm admits the same SUs as that of the centralized algorithm. Note that the x-axis of Fig. 2(a) is on a logarithm scale. Fig. 2(b) and Fig. 3(b) show that the transmit powers of the admitted SUs converge to those obtained by the centralized algorithm (as indicated by the solid lines). In Fig. 3(b), the transmit power of SU 1 after the convergence of the distributed algorithm is not important because it is not admitted for communications since $s_1 \neq 1$ in Fig. 3(a), hence SU 1 will not transmit at all.



(a)



(b)

Fig. 3. (a) Value of s_i of SUs obtained by the proposed distributed algorithm versus iteration index. Only SU 2 and SU 3 are admitted. (b) Transmit powers of SUs obtained by the proposed distributed algorithm versus iteration index. The solid lines indicate the transmit powers of the respective SUs as obtained by the centralized algorithm.

VI. CONCLUSION

In this paper, we investigated the distributed implementation of a shadowing time-scale based joint admission and power control algorithm in two-tier small cell networks. In particular, the number of small cell users that can be admitted at their outage probability specifications is maximized and their total transmit power is minimized while guaranteeing the outage probability specification of the macrocell user. As this joint admission and power control problem is NP-hard, convex relaxation is applied to obtain high quality approximate solutions. The distributed implementation is developed by applying dual decomposition technique which empowers the small cells to determine their own admissibility into the system and the

transmit powers if they are admitted.

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