

# Throughput Optimization in Wireless Networks Under Stability and Packet Loss Constraints

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**Abstract**—The problem of throughput optimization in decentralized wireless networks with spatial randomness under queue stability and packet loss constraints is investigated in this paper. Two key performance measures are analyzed, namely the effective link throughput and the network spatial throughput. Specifically, the tuple of medium access probability, coding rate, and maximum number of retransmissions that maximize each throughput metric is analytically derived for a class of Poisson networks, in which packets arrive at the transmitters following a geometrical distribution. Necessary conditions so that the effective link throughput and the network spatial throughput are stable and achievable under bounded packet loss are determined, as well as upper bounds for both cases by considering the unconstrained optimization problem. Our results show in which system configuration stable achievable throughput can be obtained as a function of the network density and the arrival rate. They also evince conditions for which the per-link throughput-maximizing operating points coincide or not with the aggregate network throughput-maximizing operating regime.

**Index Terms**—Ad hoc networks, interference, Poisson point process, stochastic geometry, queue stability, spatial throughput

## 1 INTRODUCTION

DECENTRALIZED wireless networks have recently attracted vivid interest due to their inherent simple deployment without the need for pre-existing infrastructure. They have been identified as an efficient and promising solution to enhance coverage and efficiency in future generations of cellular systems [1]. Despite their practical appeal and significant recent progress, the fundamental limits of decentralized wireless networks still remain elusive and constitute an active research area [2], [3]. One of the main research challenges is the characterization and the treatment of co-channel interference, since the behavior and the operating parameters of each transmit node affect concurrent transmissions.

Gupta and Kumar introduced in their seminal work [4] the concept of *transport capacity* as a mean to characterize the behavior of multi-hop ad hoc networks in several scenarios when the number of nodes grows to infinity. The idea behind their approach resorts to find, given a set of assumptions, a deterministic relation between link capacity

(or throughput), interference, and distance in the asymptotic regime. Even though this work and many subsequent papers (e.g. [5] and references therein) provide key results on capacity scaling laws for large number of nodes, their limitations become evident when one focuses on the performance of various PHY layer strategies, such as multiple antennas, power control, user selection, interference cancellation, and bandwidth partitioning.

In light of this fact, Weber *et al.* introduced the *transmission capacity* (TC) metric, defined as the maximum density of successful transmissions of a certain rate and subject to an outage probability constraint that can be supported per unit network area [6]. This performance metric is normally evaluated by considering a snapshot of a single-hop network, in which transmitters access the network using a slotted ALOHA protocol and node locations follow a homogeneous Poisson point process (PPP) [7]–[9]. Based on the analytical convenience of PPP, this framework allows for accurate performance evaluation of several cutting-edge communication strategies in wireless ad hoc networks, as summarized in [10] or in [11], [12], by evaluating the average performance of the network over different spatial realizations.

More recently, Andrews *et al.* proposed an extension of TC, namely *random access transport capacity* (RTC) [13], so as to capture the effect of multi-hop communication and retransmissions on the end-to-end throughput performance. Following this line of work, Vaze derived in [14] the throughput-delay-reliability tradeoff in wireless ad hoc networks with automatic repeat request (ARQ). The author evaluates how the number of hops and retransmissions of packets detected in error employing an ARQ protocol affect a revisited version of the transmission capacity metric. Interestingly, the number of multiple hops and a delay constraint, which is related to them and

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packet retransmissions, are incorporated in the modified TC metric.

Nevertheless, in all studies cited above, neither queuing delay nor packet arrival process are considered, which may lead to unstable system operation. Queue stability has been extensively studied in general settings for stochastic networks (refer to [15]) as well as in slotted ALOHA systems [16]. In [17], the authors study the interplay between stability and capacity in two-dimensional grid and ring (regular) networks. Recently, Stamatiou and Haenggi made a first step towards the combination of the stochastic geometry framework and queuing theory by analyzing the stability and the average delay of single-hop ad hoc networks [18].

In this paper, we extend the spatial throughput framework by studying single-hop networks with Poisson field of interferers and a limited number of retransmissions under maximum packet loss probability and queue stability constraints. This provides another step towards a combined approach for addressing the longtime unconsumed union between information and networking theory [19]. Specifically, a constrained maximization problem for the *effective link throughput* and the *network spatial throughput* of a random access network is cast, in which transmitters are located according to a PPP, packet inter-arrival time is geometrically distributed, and there is a bounded number of retransmissions. In both optimization problems, we are interested in determining the operating points, i.e. access probability, coding rate and maximum number of retransmissions, which lead to the highest performance subject to those constraints, given the packet arrival process and the density of transmitters in the network. Closed-form approximate solutions are then derived for both design settings as well as upper bounds on their highest achievable values. Our results show the effect of the network density and arrival rate on the network performance, indicating under which network parameters the optimal constrained performance converges to its unconstrained optimal value. Necessary conditions so that the effective link throughput and the spatial throughput are achievable under the stability and packet loss constraints are also provided. Finally, we make evident the effect of selfish and collective decisions on the network efficiency, indicating when selfish link optimization behavior can be the best choice in terms of network-wide sum throughput performance.

The rest of this paper is divided as follows. Section 2 introduces the system model and the performance metrics employed here. Section 3 presents analytical and numerical results regarding the effective link throughput optimization, while Section 4 contains the network spatial throughput analysis. Section 5 concludes this paper summarizing our results and indicating future perspectives.

## 2 SYSTEM MODEL AND BASELINE DEFINITIONS

### 2.1 Network Model

We consider a decentralized (ad hoc) wireless network, in which the spatial locations of transmitters (TXs) at each timeslot  $t \in \mathbb{N}_+$  are distributed according to a homogeneous (stationary and isotropic) point process  $\Phi_0 \subset \mathbb{R}^2$  with non-null intensity  $\lambda_0$  [TXs/m<sup>2</sup>]. Each TX is associated with one receiver (RX) and packets arrive at the buffer of TX<sub>k</sub>,  $k \in \mathcal{N}_0$ ,

according to a stochastic arrival process  $X_k(t)$ , where  $\mathcal{N}_0$  denotes the set of all TXs generated by  $\Phi_0$ . The arrival process to transmitter TX<sub>k</sub> is assumed to be stationary with an average rate  $\mu_k$  packets/slot. We assume buffers of infinite capacity and that time is slotted with slot duration equal to the packet duration.

At the end of each timeslot  $t$ , the locations of the nodes are shuffled following a high mobility random walk as proposed in [18]. Due to this particular mobility model, the displacement theorem can be applied [8, Section 1.3.3] and hence the TXs' locations in each timeslot  $t$  are generated as a different sample of the point process  $\Phi_0$ . This assumption results in independence between the nodes' positions across timeslots.<sup>1</sup>

We focus on point-to-point links, in which each TX<sub>k</sub> employs Gaussian point-to-point (G-ptp) codes and its corresponding receiver RX<sub>k</sub> treats interference as noise (IAN) [22]. The communication between TX<sub>k</sub>-RX<sub>k</sub> is considered to be successful (i.e. not in outage) using G-ptp codes and IAN decoding rule if the received signal-to-interference-plus-noise ratio (SINR) throughout the packet duration satisfies at timeslot  $t$

$$\text{SINR}_k(t) = \frac{S_{kk}(t)}{N_0 + \sum_{j \in \mathcal{N}(t) \setminus \{k\}} S_{jk}(t)} \geq \beta_k, \quad (1)$$

where  $\mathcal{N}(t) \subseteq \mathcal{N}_0$  refers to the subset of active TXs in timeslot  $t$ ,  $S_{xy}(t)$  is the total received power at RX<sub>y</sub> from TX<sub>x</sub> and  $N_0$  is the power (variance) of the independent additive white complex Gaussian noise of zero mean. The SINR threshold  $\beta_k$  required by RX<sub>k</sub> to successfully decode the packets is a system parameter, which depends - among others - on the coding rate, the modulation scheme, and the target bit error rate (BER). We assume that each TX<sub>k</sub> employs G-ptp codes with rate  $R_k$  [bits/s/Hz], which can be related using the Shannon formula with the minimum SINR as<sup>2</sup>  $R_k = \log_2(1 + \beta_k)$ .

Automatic repeat-request (ARQ) protocol [24] is considered, hence the success or failure (outage) of the packet detection at RX is reported back to TX through an error and delay-free control channel. In that case, the undelivered packet returns to the head-of-line (HOL) of the queue, waiting to be retransmitted in the next medium access. Assuming that a packet can be retransmitted through the TX<sub>k</sub>-RX<sub>k</sub> link at most  $m_k$  times, then there are two possible outcomes for packet departure from the queue of TX<sub>k</sub>, namely (i) it is either correctly received or (ii) it is not successfully received after  $1 + m_k$  attempts and then dropped from the queue, declaring a packet loss event. Hence, the packet loss probability for TX<sub>k</sub>-RX<sub>k</sub>, denoted

1. The high mobility random walk is somehow an artificial mobility model and practical scenarios should consider correlation between events that occur in different timeslots [20], [21]. Nevertheless, such an approach is hard or impossible to lead to closed-form, analytical expressions due to the coupling between service rate and interference process. For that, in this work, we employ the high mobility random walk model in order to derive neat closed-form expressions, which in turn are still able to provide valuable insights on the network performance.

2. The timeslot duration is considered to be large enough to sustain rates arbitrarily close to the channel capacity, which implies that the code length goes to infinity. The effect of finite-length codes on the error probability in spatial wireless networks is studied in [23].

$P_{\ell,k}$ , is a function of the number of allowed retransmissions and the outage probability, i.e.  $P_{\ell,k} = f(P_{o,k}, m_k)$ , where the outage probability is given by  $P_{o,k} = \mathbb{P}[\text{SINR}_k < \beta_k]$ .

### 2.2 Queue Stability

Assuming here a single-server discrete-time queuing system, the backlog  $Q_k(t)$  (queue length) for  $\text{TX}_k$  is evolving for  $t \in \{0, 1, 2, \dots\}$  as [15]:

$$Q_k(t + 1) = \max[Q_k(t) - Y_k(t), 0] + X_k(t), \quad (2)$$

where  $\{X_k(t)\}_{t=0}^{\infty}$  and  $\{Y_k(t)\}_{t=0}^{\infty}$  are the arrival and the server process at  $\text{TX}_k$  in timeslot  $t$  and the initial queue lengths  $\{Q_k(0)\}$  are chosen independently across TXs according to some probability distribution. Note that packet arrivals and channel access events are independent across sources and slots.

For the definition of queue stability, we resort to [25].

**Definition 1 (stability).** A multidimensional stochastic process (not necessarily Markovian)  $\mathbf{Q}(t) = (Q_1(t), \dots, Q_1(t))$  is stable if for  $\mathbf{x} \in \mathbb{N}_0^M$  the following holds

$$\lim_{t \rightarrow \infty} \mathbb{P}[\mathbf{Q}(t) < \mathbf{x}] = F(\mathbf{x}) \quad \text{and} \quad \lim_{\mathbf{x} \rightarrow \infty} F(\mathbf{x}) = 1, \quad (3)$$

where  $F(\mathbf{x})$  is the limiting distribution function and  $\mathbf{x} \rightarrow \infty$  means that  $x_k \rightarrow \infty, \forall k$ . If a weaker condition holds, namely,

$$\lim_{\mathbf{x} \rightarrow \infty} \liminf_{t \rightarrow \infty} \mathbb{P}[\mathbf{Q}(t) < \mathbf{x}] = 1, \quad (4)$$

then the process is called substable (tight or bounded in probability).

The queue stability evidently depends on both  $\{X_k(t)\}_{t=0}^{\infty}$  and  $\{Y_k(t)\}_{t=0}^{\infty}$ . While the former is an input parameter that the network operator cannot always control, the latter is determined by the medium access (MAC) protocol, the retransmission policy, and the probability that a packet is successfully received during a transmission attempt. Such a success probability is in fact a physical layer figure, which in turn is related to the decoding strategy, co-channel interference, noise power, and desired signal strength.

### 2.3 Performance Metrics

Based on the system model presented above, we define the performance metrics of interest, which are the effective throughput of a point-to-point link and the spatial throughput of the network.

**Definition 2 (effective link throughput).** Given that the network is in steady state, the effective link throughput of a given link  $\text{TX}_k\text{-RX}_k$ , denoted by  $\mathcal{R}_k$  and measured in [bits/s/Hz], is defined as

$$\mathcal{R}_k = (1 - P_{\ell,k}) p_k \rho_k \frac{R_k}{1 + \bar{m}_k}, \quad (5)$$

where  $\rho_k$  is the probability that the queue of  $\text{TX}_k$  is not empty in a given timeslot,  $p_k$  is the probability that  $\text{TX}_k$  is granted to access the radio channel in a given timeslot, and  $\bar{m}_k$  is the average number of packet retransmissions.

**Definition 3 (network spatial throughput).** Given that the network is in steady state, the spatial throughput, denoted by  $S$  and measured in [bits/s/Hz/m<sup>2</sup>], is defined as the sum of the effective link throughputs  $\mathcal{R}_k \forall k \in \mathcal{N}_0$  divided by the total

network area  $|A|$  [m<sup>2</sup>] of Borel subset  $A$  where the points of process  $\Phi_0$  are distributed, i.e.

$$S = \frac{1}{|A|} \sum_{k \in \mathcal{N}_0} \mathcal{R}_k. \quad (6)$$

In the following sections, we use these definitions to assess and optimize the performance of decentralized networks where the transmitters' locations are spatially distributed according to a homogeneous Poisson point process (PPP). Specifically, in Section 3, we study how the typical link tunes selfishly its access probability, its maximum number of retransmissions, and the SINR target so as to maximize its own individual effective link throughput. In Section 4, on the other hand, we take on a different approach by considering the spatial network throughput, formulating a network-wide optimization problem, and showing under which conditions (locally) selfish behavior also maximizes the aggregate throughput.

## 3 EFFECTIVE LINK THROUGHPUT OPTIMIZATION IN POISSON NETWORKS

In this section, we aim at computing the maximum effective throughput supported by a link such that the packet loss probability is bounded by a certain maximum value and queue stability is guaranteed. Specifically, we show which design parameters achieve maximum performance and then analyze how the effective throughput is related to the network density and the arrival rate.

### 3.1 Scenario Description

Let  $\Phi_0$  be a homogeneous Poisson point process of intensity  $\lambda_0$  [TXs/m<sup>2</sup>] distributed over the infinite plane, i.e. the point process  $\Phi_0$  is analyzed in  $\mathbb{R}^2$  and therefore the number  $K$  of TXs over the network tends to infinity. For convenience, we describe here a scenario in which there are  $K$  TXs distributed over the network area but always keeping in mind that  $K \rightarrow \infty$ .

Let us assume that at the beginning of each timeslot  $t$  every  $\text{TX}_k$  with  $k \in \mathcal{N}_0$  is granted access to the network with probability  $p_k$  independently of all other nodes (slotted ALOHA) and its queue state. We define a vector  $\mathbf{p} = (p_1, \dots, p_K) \in [0, 1]^K$  associated with the fixed channel access probability of  $\text{TX}_k, k = 1, \dots, K$  with  $K = |\mathcal{N}_0|$ . Furthermore, if the queue system of all TXs is in steady state, we can compute the probability that  $\text{TX}_k$  does not have any packet in its buffer to send as  $1 - \rho_k$ , for  $k \in \mathcal{N}_0$ , and then we can similarly define the vector  $\boldsymbol{\rho} = (\rho_1, \dots, \rho_K) \in [0, 1]^K$ . The probability  $\rho_k$  is related to the offered load of  $\text{TX}_k$ 's queue, as discussed next.

Let us consider that every  $\text{TX}_k$  is subject to independent geometrical arrivals with rate  $0 \leq \mu_k \leq 1$ , allowing us to define the set of arrival rates  $\boldsymbol{\mu} = (\mu_1, \dots, \mu_K) \in [0, 1]^K$ . Assuming that the server process has finite average  $\mathbb{E}[Y_k(t)] = \theta_k \leq 1$ , which is a function of the access probability, the outage probability and the number of allowed retransmissions, i.e.  $\theta_k = f(p_k, P_{o,k}, m_k)$ , the non empty state probability (or the load of the queue system),  $\rho_k$  is defined as [26]:

$$\rho_k \triangleq \frac{\mu_k}{\theta_k}. \quad (7)$$



From this, we can clearly see that  $\rho$  is a function of  $\mathbf{p}$ . In the steady state, the probabilities given by the vector  $\rho$  are also fixed regardless of which TXs are granted access to the network in a given timeslot. Once again it is important to remember that we consider here the high mobility random walk such that the nodes' position in every timeslot can be viewed as a different and independent realization of the point process  $\Phi_0$ . Therefore, we can identify two independent events related to every TX $_k$  in a given timeslot  $t$  assuming that the network has already reached its steady state: (i) access the network with a fixed probability  $p_k$  and (ii) have an empty queue with probability  $1 - \rho_k$ .

Based on the above facts and the homogeneity of the PPP  $\Phi_0$ , we can characterize the point process of the active TXs in network (i.e. the nodes that access the network and have a packet to transmit in their queues) once the steady state is achieved, by applying two thinning transformations [7, Section 1.7] associated with the events described above. Let us denote by  $\Phi_a$  the point process byproduct of a thinning transformation of  $\Phi_0$  [7] related to the network access defined by the vector of probabilities  $\mathbf{p}$ . Hence we use Theorems 1.3 and 2.3 from [7] to verify that  $\Phi_a$  also forms a homogeneous PPP with intensity  $\lambda_a = \bar{p} \lambda_0$ , where  $\bar{x} = \|\mathbf{x}\|_1 / K$  with  $\|\mathbf{x}\|_1$  is the  $L^1$ -norm of a vector  $\mathbf{x}$ . It is important to note that  $\bar{p}$  can be interpreted as the average access transmit probability.

Note that, in the steady state, the probability that an empty queue event occurs in a given slot  $t$  for every TX $_k$ ,  $\forall k \in \mathcal{N}_0$ , is independent of the event of TX $_k$  being granted to access the network in that slot, even though the probability  $\rho_k$  is a function of  $p_k$ . Knowing this, we can characterize the process of the actual concurrent transmissions  $\Phi$ , which is also a homogeneous PPP, as a thinning transformation of  $\Phi_a$  in accordance with the probabilities characterized by  $\rho$ . Then, we proceed as before evaluating the intensity of the process  $\Phi$  as  $\lambda = \bar{p} \bar{\rho} \lambda_0$ .

### 3.2 Analytical Results

We focus on a typical link TX $_0$ –RX $_0$  such that RX $_0$  is placed at the origin of  $\Phi_0$  and TX $_0$  is located at fixed distance  $d$  from it. Packets arrive at TX $_0$  queue system with rate  $\mu_0$ , according to a geometric distribution. We calculate the access probability  $p_0$ , the coding rate  $R_0$ , and the maximum number of retransmissions  $m_0$ , which lead to the maximum effective stable throughput (see Definition 2) for the typical link TX $_0$ –RX $_0$ .

We consider the received power at RX $_y$  due to TX $_x$  at time slot  $t$ ,  $S_{xy}(t)$ , is composed by two components: (i) distance-dependent pathloss  $\ell_{xy}(t)$ , and (ii) small-scale fading channel gain  $g_{xy}(t)$ , assumed to be i.i.d. (across time and space) random variable and constant during timeslot  $t$ . The transmit power is normalized to one without loss of generality. Then, if the noise power is negligible compared to the aggregate interference (interference-limited regime) and a standard power law pathloss attenuation  $\ell_{xy}(t) = r_{xy}^{-\alpha}(t)$  is used, with pathloss exponent  $\alpha > 2$  and  $r_{xy}(t)$  being the distance between TX $_x$  and RX $_y$  during timeslot  $t$ , the signal-to-interference ratio (SIR) is given by

$$\text{SIR}_0(t) = \frac{g_{00}(t) d^{-\alpha}}{\sum_{j \in \mathcal{N}(t)} g_{j0}(t) (r_{j0}(t))^{-\alpha}}, \quad (8)$$

and the success probability is given by  $\mathbb{P}[\text{SIR}_0 \geq \beta_0]$ , where  $\beta_0$  is the SIR threshold required by TX $_0$ –RX $_0$  in order to sustain a rate of  $R_0 = \log_2(1 + \beta_0)$  with arbitrarily low error probability.

We derive here the outage probability for the Rayleigh fading case for exposition convenience, i.e.  $g$  is an exponential random variable with unit mean. Our results can be easily extended to other general fading distributions; however this is out of the scope of this work. Moreover, the time index is dropped whenever the quantities and the results are independent of the timeslot  $t$ , i.e. in the steady state.

**Proposition 1.** *Given that all TXs have stable queues and that the network is in steady state, the outage probability  $P_{o,0}$  experienced by TX $_0$ –RX $_0$  is given by*

$$P_{o,0} = 1 - e^{-\bar{p} \bar{\rho} \lambda_0 \kappa d^2 \beta_0^{2/\alpha}}, \quad (9)$$

where  $\kappa = \pi \Gamma(1 + 2/\alpha) \Gamma(1 - 2/\alpha)$ . In addition, if the maximum acceptable packet loss probability is bounded by a threshold  $\epsilon$ , the following inequality has to be satisfied:

$$P_{o,0}^{1+m_0} \leq \epsilon. \quad (10)$$

**Proof.** The proof of (9) follows the results presented in [8], noting that  $\bar{p}$  and  $\bar{\rho}$  are constants when all TXs have stable queues and the network is in steady state. To prove (10), we use the fact that packets not successfully decoded by RX $_0$  can be retransmitted up to  $m_0$  times before being dropped and that the outage events are independent across timeslots, yielding that the packet loss probability is  $P_{\ell,0} = P_{o,0}^{1+m_0}$ .  $\square$

We now proceed with obtaining the probability that the buffer of a typical TX $_0$  is empty using similar arguments as in [18], under the assumption that a high mobility random walk model is considered and that the point process  $\Phi$  is studied in  $\mathbb{R}^2$ . If we consider that the system is in steady state and recall that the queue of the typical TX is subject to i.i.d. packet arrivals with probability  $\mu_0$  and i.i.d. departures with probability  $\theta_0$ , then the offered load is  $\rho_0 = \mu_0/\theta_0$ . Using elements from the theory of G/G/1 queues, the probability that the queue is empty is shown to be  $\max[0, 1 - \rho_0]$  [26]. Furthermore, from the definition of  $\rho_0$  and assuming stable queues for all TXs, we provide the following result.

**Proposition 2 (service rate).** *Given that all TXs have stable queues and the network is in steady state, the service rate  $\theta_0$  of a typical link is given by*

$$\theta_0 = p_0 \frac{e^{-\bar{p} \bar{\rho} \lambda_0 \kappa d^2 \beta_0^{2/\alpha}}}{1 - \left(1 - e^{-\bar{p} \bar{\rho} \lambda_0 \kappa d^2 \beta_0^{2/\alpha}}\right)^{1+m_0}}. \quad (11)$$

**Proof.** We first recall that the medium access process is independent of the outage events, which in turn are independent across timeslots. Therefore, we have that

$$\theta_0 = \frac{p_0}{1 + \bar{m}_0}, \quad (12)$$

where  $1 + \bar{m}_0$  is the average number of transmission attempts available for a packet arriving at TX $_0$ , which is

$$1 + \bar{m}_0 = 1 + P_{o,0} + P_{o,0}^2 + \dots + P_{o,0}^{m_0} = \sum_{k=0}^{m_0} P_{o,0}^k = \frac{1 - P_{o,0}^{1+m_0}}{1 - P_{o,0}}. \quad (13)$$

The proof is concluded using (9) → (13) → (12). □

The above proposition provided the server rate at TX<sub>0</sub> assuming that its queue is stable. We derive now a sufficient condition which guarantees queue stability at TX<sub>0</sub>.

**Proposition 3 (queue stability).** *Given that all TXs excluding TX<sub>0</sub> have stable queues and that the network is in steady state, a sufficient condition for queue stability at TX<sub>0</sub> is given by*

$$\mu_0 < p_0 \left[ \sum_{i=1}^{1+m_0} \binom{1+m_0}{i} (-1)^{i+1} e^{-\bar{p}\lambda_0(i-1)\kappa d^2\beta_0^{2/\alpha}} \right]^{-1}. \quad (14)$$

**Proof.** To provide a sufficient condition for stability for the queue of TX<sub>0</sub>, we first assume the worst case scenario for interference, namely all TXs granted access to the network transmit packets regardless of their backlog state (see *dominant network* [16]). In other words, transmitters with empty queues make dummy transmissions, yielding  $\bar{\rho} = 1$  and that the density of active transmitters is  $\lambda = \bar{p}\lambda_0$ . Let  $\theta_{0,\text{dom}}$  denote the server rate of TX<sub>0</sub> for the dominant network configuration. Based on the fact that the arrival and server processes are jointly ergodic and stationary (refer to [18]), we can use the inequality  $\mu_0 < \theta_{0,\text{dom}}$  as a sufficient condition for the stability of the typical link [26]. Then, applying these statements into (11) yields

$$\mu_0 < \theta_{0,\text{dom}} = p_0 \frac{e^{-\bar{p}\lambda_0\kappa d^2\beta_0^{2/\alpha}}}{1 - \left(1 - e^{-\bar{p}\lambda_0\kappa d^2\beta_0^{2/\alpha}}\right)^{1+m_0}}. \quad (15)$$

To obtain (14), we manipulate the binomial expansion of the denominator of (15), which concluded the proof. □

**Remark.** From Definition 1, we can say that, when  $\rho_0 = \frac{\mu_0}{\theta_0} \rightarrow 1$ , the TX<sub>0</sub>'s queue is in the boundary of stability.

Before presenting the optimization problem that is the main topic of this section, we rewrite the effective link throughput formulation stated in Definition 2 as:

$$\begin{aligned} \mathcal{R}_0 &= (1 - P_{\ell,0}) p_0 \rho_0 \frac{R_0}{1 + \bar{m}_0} \\ &= (1 - P_{o,0}^{1+m_0}) p_0 \rho_0 \frac{R_0 (1 - P_{o,0})}{1 - P_{o,0}^{1+m_0}} \\ &= p_0 \rho_0 \log_2(1 + \beta_0) e^{-\bar{p}\lambda_0\kappa d^2\beta_0^{2/\alpha}}. \end{aligned} \quad (16)$$

Combining the above propositions with (16), we formulate the optimization problem which provides the highest effective link throughput the typical link TX<sub>0</sub>–RX<sub>0</sub> can achieve while its packet loss probability is bounded by a

maximum value  $\epsilon$  and its queue is stable as follows<sup>3</sup>:

$$\begin{aligned} \max_{(p_0, \beta_0, m_0)} \quad & p_0 \rho_0 \log_2(1 + \beta_0) e^{-\bar{p}\lambda_0\kappa d^2\beta_0^{2/\alpha}} \\ \text{s.t.} \quad & \left(1 - e^{-\bar{p}\lambda_0\kappa d^2\beta_0^{2/\alpha}}\right)^{1+m_0} \leq \epsilon, \\ & \theta_0 = p_0 \frac{e^{-\bar{p}\lambda_0\kappa d^2\beta_0^{2/\alpha}}}{1 - \left(1 - e^{-\bar{p}\lambda_0\kappa d^2\beta_0^{2/\alpha}}\right)^{1+m_0}}, \\ & \theta_0 \geq p_0 \left[ \sum_{i=1}^{1+m_0} \binom{1+m_0}{i} (-1)^{i+1} e^{-\bar{p}\lambda_0(i-1)\kappa d^2\beta_0^{2/\alpha}} \right]^{-1} > \mu_0. \end{aligned}$$

The above optimization problem is in general non-convex, hence it is hard to obtain an analytical solution. In order to gain some insight, we propose an approximate closed-form solution to determine the maximum constrained effective link throughput  $\mathcal{R}_0^*$  by observing some properties of the problem.

**Proposition 4 (Highest constrained  $\mathcal{R}_0$ ).** *Given that all TXs excluding TX<sub>0</sub> have stable queues, the packet loss constraint  $\epsilon$  has a small value and the system has reached its steady state, and assuming that the number of retransmissions  $m_0$  is a non negative real number, the highest effective throughput  $\mathcal{R}_0^*$  achieved by the typical link TX<sub>0</sub>–RX<sub>0</sub> under queue stability and bounded packet loss can be approximated by*

$$\mathcal{R}_0^* \approx \mu_0 (1 - \epsilon) \log_2 \left( 1 + \left( \frac{-\ln(\mu_0 (1 - \epsilon))}{\bar{p}\lambda_0\kappa d^2} \right)^{\alpha/2} \right), \quad (17)$$

where the system parameters  $(p_0^*, \beta_0^*, m_0^*)$  yielding the approximated  $\mathcal{R}_0^*$  are given by

$$\begin{aligned} p_0^* &= 1, \\ \beta_0^* &= \left( \frac{-\ln(\mu_0 (1 - \epsilon))}{\bar{p}\lambda_0\kappa d^2} \right)^{\alpha/2}, \\ m_0^* &= \frac{1}{\log_\epsilon(1 - \mu_0(1 - \epsilon))} - 1. \end{aligned}$$

**Proof.** The optimal values or functions of them are denoted below using \*. Note first that the optimal solution should lie on the stability boundary (i.e.  $\rho_0^* = \theta_0^*/\mu_0 \rightarrow 1$ ), indicating that whenever TX<sub>0</sub> is granted access to the medium, it has a packet to transmit. As the packet loss constraint  $\epsilon$  is a small number, we can make the following approximation  $P_{\ell,0}^* = P_{o,0}^{*1+m_0^*} \approx \epsilon$  for the optimal solution (i.e.  $P_{\ell,0}$  is approximated by its maximum acceptable value). From these observations, using equations (12) and (13), we have the following relation

$$\mu_0 \approx \theta_0^* = p_0^* \frac{1 - \epsilon^{\frac{1}{1+m_0^*}}}{1 - \epsilon} \Rightarrow p_0^* \left(1 - \epsilon^{\frac{1}{1+m_0^*}}\right) \approx \mu_0(1 - \epsilon). \quad (18)$$

3. It is interesting to say that in some applications there is a requirement of minimum coding rate  $R_0$  and then an additional constraint regarding the threshold  $\beta_0$  must be included. Although a different optimization formulation for that case may be found, it will not outperform the solution without the minimum rate constraint, which in fact provides an upper bound to the new problem.

We also manipulate the outage constraint (10) knowing that  $P_{o,0}^* = \epsilon^{\frac{1}{1+m_0^*}}$ , which yields

$$\epsilon \approx \left(1 - e^{-\bar{\rho} \bar{p} \lambda_0 \kappa d^2 \beta_0^{*2/\alpha}}\right)^{1+m_0^*}. \quad (19)$$

Then we can combine (18) and (19) by isolating the term  $1 - \epsilon^{\frac{1}{1+m_0^*}}$  to obtain the following relation

$$1 - \epsilon^{\frac{1}{1+m_0^*}} \approx \frac{\mu_0}{p_0^*} (1 - \epsilon) \approx e^{-\bar{\rho} \bar{p} \lambda_0 \kappa d^2 \beta_0^{*2/\alpha}},$$

from where, after algebraic manipulations, we compute the SIR threshold  $\beta_0^*$  that leads to the approximation of the highest effective throughput as

$$\beta_0^* \approx \left( \frac{\ln(p_0^*) - \ln(\mu_0(1 - \epsilon))}{\bar{\rho} \bar{p} \lambda_0 \kappa d^2} \right)^{\alpha/2}. \quad (20)$$

Recalling that  $R_0^* = \log_2(1 + \beta_0^*)$ , we then apply (18) and (20) into (16), which results in

$$\mathcal{R}_0^* \approx \mu_0 (1 - \epsilon) \log_2 \left( 1 + \left( \frac{\ln(p_0^*) - \ln(\mu_0(1 - \epsilon))}{\bar{\rho} \bar{p} \lambda_0 \kappa d^2} \right)^{\alpha/2} \right). \quad (21)$$

In order to optimize (21), the access probability  $p_0^*$  should be made as large as possible, resulting in  $p_0^* = 1$ , which proves (17). To conclude the proof, we apply  $p_0^* = 1$  into (20) and (18), obtaining then the design parameters that maximize the approximated  $\mathcal{R}_0^*$ .  $\square$

From the equations presented in Proposition 4, we can state interesting properties for the proposed approximation of the optimal link effective throughput as follows.

**Corollary 1.** *The link throughput  $\mathcal{R}_0^*$  and the system parameters ( $p_0^*$ ,  $\beta_0^*$ ,  $m_0^*$ ) stated in Proposition 4 have the following properties:*

- (a)  $\mathcal{R}_0^*$  is a concave function of  $\mu_0 \in [0, 1]$  and a monotonically decreasing function of  $\lambda_0 > 0$ ,  $\bar{p} \in [0, 1]$  and  $\bar{\rho} \in [0, 1]$ ;
- (a2) the arrival rate  $\mu_0^* \in [0, 1]$  that maximizes  $\mathcal{R}_0^*$  can be found as the  $\mu_0 \in [0, 1]$  solution<sup>4</sup> to the derivative equation:  $d\mathcal{R}_0^*/d\mu_0 = 0$ ;
- (b)  $p_0^*$  has a constant value regardless of  $\mu_0 \in [0, 1]$ ,  $\lambda_0 > 0$ ,  $\bar{p} \in [0, 1]$  and  $\bar{\rho} \in [0, 1]$ ;
- (c)  $\beta_0^*$  is a monotonically decreasing function of  $\mu_0 \in [0, 1]$ ,  $\lambda_0 > 0$ ,  $\bar{p} \in [0, 1]$  and  $\bar{\rho} \in [0, 1]$ ;
- (d)  $m_0^*$  is a monotonically decreasing function of  $\mu_0 \in [0, 1]$  and not affected by  $\lambda_0 > 0$ ,  $\bar{p} \in [0, 1]$  and  $\bar{\rho} \in [0, 1]$ .

The proof for this corollary is straightforward by inspection of equations (17) and (18), and for this reason it is omitted here. Further discussions of such properties will be provided when analyzing the numerical results in Section 3.3. Next we obtain further interesting results derived from Proposition 4.

4. Note that a closed-form solution is not possible in this case, but we can always resort to numerical evaluation to find such a maximum.

**Proposition 5 (upper bound of  $\mathcal{R}_0^*$ ).** *An upper bound on effective link throughput  $\mathcal{R}_0^*$  stated in Proposition 4 is*

$$\mathcal{R}_0^* \leq \mathcal{R}_{0,\text{up}} = \log_2(1 + \beta_{\text{up}}) e^{-\bar{\rho} \bar{p} \lambda_0 \kappa d^2 \beta_{\text{up}}^{2/\alpha}}, \quad (22)$$

where  $\beta_{\text{up}}$  is found as the value of  $\beta_0$  solution to

$$\beta_0 = 2 \bar{\rho} \bar{p} \lambda_0 \kappa d^2 \beta_0^{2/\alpha} (1 + \beta_0) \ln(1 + \beta_0). \quad (23)$$

**Proof.** We first use the fact that the effective throughput obtained by the unconstrained optimization of eq. (16) is always an upper bound to the constrained optimization given by (17). The unconstrained objective function (see eq. (16)) is maximized for  $p_0 = 1$  and  $\rho_0 \rightarrow 1$ . Furthermore, it can be easily shown that (16) is a concave function of  $\beta_0$  with  $\beta_0 > 0$ , hence taking the derivative  $d\mathcal{R}_0/d\beta_0$  and after some manipulations, its optimal value is given by (23). To conclude the proof, we put this optimal value into (16), which yields (22).  $\square$

**Theorem 1 (necessary condition for  $\mathcal{R}_0^*$ ).** *Given that all TXs except  $\text{TX}_k$  have stable queues, the packet loss constraint  $\epsilon$  has a small value and the network is in steady state, a necessary condition so that the effective throughput  $\mathcal{R}_k$ ,  $k \in \mathcal{N}_0$ , is achievable with bounded packet loss probability and queue stability is given by*

$$\mathcal{R}_k < \mu_k (1 - \epsilon) \log_2 \left( 1 + \left( \frac{-\ln(\mu_k (1 - \epsilon))}{\bar{\rho}_{\mathcal{N}_0 \setminus \{k\}} \bar{p}_{\mathcal{N}_0 \setminus \{k\}} \lambda_0 \kappa d^2} \right)^{\alpha/2} \right), \quad (24)$$

where  $\mu_k$  is the arrival rate at  $\text{TX}_k$  and the subindex  $\mathcal{N}_0 \setminus \{k\}$  indicates that the averages  $\bar{\rho}$  and  $\bar{p}$  do not take into account link  $k$ .

**Proof.** First, we use the fact that, in the steady state, the stochastic processes that determine the network behavior are stationary and isotropic over timeslots and links. Then, using properties of Palm distribution and Slivnyak's theorem [7] to evaluate the statistical properties of every link  $\text{TX}_k$ - $\text{RX}_k$  with  $k \in \mathcal{N}_0$  based on one typical link  $\text{TX}_0$ - $\text{RX}_0$ , the index 0 can be exchanged by  $k$  in (17) to obtain the maximum effective throughput  $\mathcal{R}_k^*$  supported by  $\text{TX}_k$ - $\text{RX}_k$  that satisfies the stability and packet loss constraints. Note that the average  $\bar{\rho}$  and  $\bar{p}$  are related to the interfering TXs that are active in a given network realization and therefore the node in study, i.e.  $\text{TX}_k$  should be excluded from the computation of such averages. Moreover,  $\bar{\rho}$  and  $\bar{p}$  have constant values since the network is assumed to have reached its steady state and all potential interfering TXs are assumed to have stable queues. To conclude this proof, we use the fact that  $\mathcal{R}_k^*$  is by definition the highest possible effective throughput under the packet loss and queue stability constraints and then every throughput  $\mathcal{R}_k$  that is subject to the same constraints should be lower than that maximum value.  $\square$

**Remark.** It is worth noting that the theorem states a necessary condition, but not sufficient. This means that it is possible to have effective throughput  $\mathcal{R}_k < \mathcal{R}_k^*$  when the queue system of  $\text{TX}_k$  is unstable and/or the packet loss probability exceeds  $\epsilon$ . Moreover, this maximum value



$\mathcal{R}_k^*$  is surprisingly only a function of the arrival rate  $\mu_k$  at  $\text{TX}_k$  and the network characteristics, and it does not depend on the specific system parameters employed to achieve it. In other words,  $\mathcal{R}_k^*$  can be viewed as a limit of the system and the design setting dictates how to achieve it.

Finally, we provide some consequent results for a system operating with optimal system parameters.

**Corollary 2.** *If all  $\text{TX}_k$ – $\text{RX}_k$  links with  $k \in \mathcal{N}_0$  employ the optimal strategy given by Proposition 4, then  $\bar{\rho}_{\mathcal{N}_0 \setminus \{k\}} \rightarrow 1$  and  $\bar{\rho}_{\mathcal{N}_0 \setminus \{k\}} = 1$ , reducing the necessary condition given by Theorem 1 to*

$$\mathcal{R}_k < \mu_k (1 - \epsilon) \log_2 \left( 1 + \left( \frac{-\log(\mu_k (1 - \epsilon))}{\lambda_0 \kappa d^2} \right)^{\alpha/2} \right). \quad (25)$$

**Corollary 3.** *Let each design choice  $(p_k, \beta_k, m_k)$  be the individual strategy profile of a game amongst  $k \in \mathcal{N}_0$  links (players or agents) distributed over the network and effective link throughput  $\mathcal{R}_k$  be the utility function of each link  $\text{TX}_k$ – $\text{RX}_k$ . Then, if all  $k$  links employ the optimal individual design described in Proposition 4, the game is in a strict Nash equilibrium [27], i.e. an individual change cannot increase its own utility function when the other links continue using the same strategy.*

**Corollary 4.** *If a zero packet loss probability is required ( $\epsilon = 0$ ), then the maximum number of retransmissions  $m_k^* \rightarrow \infty, \forall k \in \mathcal{N}_0$ .*

**Corollary 5.** *If all  $\text{TX}_k$ – $\text{RX}_k$  links with  $k \in \mathcal{N}_0$  employ the optimal strategy described in Proposition 4 and all TXs are subject to the same arrival rates  $\mu_k = \mu \forall k \in \mathcal{N}_0$  (symmetric case), then the spatial throughput  $S_{\text{ind}}^*$  for this scenario is computed as*

$$S_{\text{ind}}^* = \lambda_0 \mu (1 - \epsilon) \log_2 \left( 1 + \left( \frac{-\log(\mu (1 - \epsilon))}{\lambda_0 \kappa d^2} \right)^{\alpha/2} \right). \quad (26)$$

The proofs of these corollaries are straightforward and therefore omitted here.

### 3.3 Numerical Results

In this section, we use the analytical expressions previously derived to provide numerical results that help us to have a better understanding of the effective link throughput behavior as a function of the network parameters. These results are also used to illustrate the properties of the approximated solution to the link throughput optimization previously derived as well as to assess the tightness of our approximation.<sup>5</sup>

The design setting parameters employed by the typical link in order to maximize its effective throughput (see Proposition 4) are shown in Table 1 for different pairs of network densities  $\lambda_0$  and arrival rates  $\mu_0$ , which are the input parameters. Table 1 also contains the optimal values computed using Proposition 4 and its upper bound as stated in Proposition 5.

5. The numerical analysis carried out here can be extended to any link of the network, as previous stated.

TABLE 1  
Optimal Effective Throughput Design Setting of  $\text{TX}_0$ – $\text{RX}_0$  for  $\bar{\rho} = 1, \bar{\rho} \rightarrow 1, \alpha = 4, d = 1$ , and  $\epsilon = 0.02$

$(\lambda_0, \mu_0)$	$(p_0^*, R_0^*, 1 + m_0^*)$	$\mathcal{R}_0^*$	$\mathcal{R}_{0,\text{up}}$
(0.1, 0.2)	(1, 3.57, 17.9)	0.700	0.865
(0.1, 0.8)	(1, 0.31, 2.6)	0.246	0.865
(0.5, 0.2)	(1, 0.52, 17.9)	0.102	0.102
(0.5, 0.8)	(1, 0.014, 2.6)	0.011	0.102

First, we analyze the scenario where  $\lambda_0 = 0.1$  [TXs/m<sup>2</sup>] and  $\mu_0 = 0.2$ . To achieve the highest effective throughput under packet loss and queue stability constraints,  $\text{TX}_0$  should set its rate equal to  $R_0^* = 3.57$  [bits/s/Hz] and have  $m_0^* = \lceil 16.9 \rceil$  possible retransmission attempts. These numbers show that  $\text{TX}_0$  communicates with high coding rate, which increases the chance that the transmitted packet is not correctly decoded by  $\text{RX}_0$  (outage event), thus a very large number of retransmissions should be allowed so that the packet loss constraint is not violated. By employing this setting,  $\text{TX}_0$ – $\text{RX}_0$  can reach an effective throughput of  $\mathcal{R}_0^* = 0.700$ , which is relatively close to its upper bound  $\mathcal{R}_{0,\text{up}} = 0.865$ , indicating some flexibility for the possible feasible solutions of our constrained optimization problem.

Second, we turn our attention to the scenario where the arrival rates are more frequent, namely  $\mu_0 = 0.8$ , still considering  $\lambda_0 = 0.1$ . From Table 1, one can clearly see that the effective throughput  $\mathcal{R}_0^*$  decreases almost 65% as compared to the case where  $\mu_0 = 0.2$ , while the upper bound remains the same. This effective throughput is achieved when both coding rate and number of retransmissions heavily decrease, indicating that the combination of  $R_0^*$  and  $m_0^*$  is limited by the queue stability. In other words, when  $\mu_0$  is large, the outage events need to happen less frequently, decreasing the rate as well as the possible number of retransmissions so that both stability and packet loss requirements are respected.

In contrast, when the network has a density  $\lambda_0 = 0.5$  and  $\text{TX}_0$  experiences an arrival rate of  $\mu_0 = 0.2$ , the effective link throughput  $\mathcal{R}_0^*$  achieves its upper bound  $\mathcal{R}_{0,\text{up}}$ . This indicates that low values of  $\mu_0$  do not impose a strict restriction to the feasible design options for the density  $\lambda_0$ . Hence the highest possible effective throughput can be reached by decreasing the coding rate  $R_0^*$  while the number of retransmissions can be still high, without violating the stability constraint. In any case, even though the effective throughput  $\mathcal{R}_0^*$  is very close to its upper bound when  $\lambda_0 = 0.5$ , its value is much lower when  $\lambda_0 = 0.1$ , which evinces the harmful effects of the co-channel interference, i.e. the higher the density of active links, the lower the  $\text{TX}_0$ – $\text{RX}_0$  effective throughput.

Finally we study the case where a dense network with high rate of arrivals is considered, verifying the substantial loss of the effective throughput  $\mathcal{R}_0^*$ . As expected, the value of  $\mathcal{R}_0^*$  presents a significant gap to its upper bound  $\mathcal{R}_{0,\text{up}}$ , indicating that the  $\text{TX}_0$ – $\text{RX}_0$  performance is severely limited by both the interference level observed by  $\text{RX}_0$  and the restricting choice of feasible solutions due to the stability constraint. It is worth noticing that some of these facts

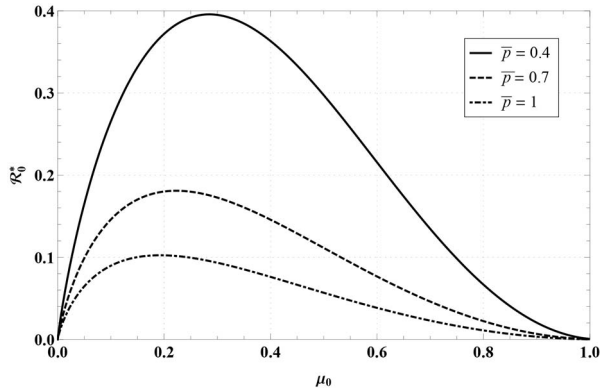


Fig. 1. Optimal effective link throughput  $\mathcal{R}_0^*$  of the typical link  $\text{TX}_0\text{--RX}_0$  (see Proposition 4) versus its arrival rate  $\mu_0$  for different  $\bar{p}$  and considering  $\alpha = 4$ ,  $d = 1$ ,  $\bar{p} \rightarrow 1$ ,  $\lambda_0 = 0.5$ , and  $\epsilon = 0.02$ .

have been already predicted by Corollary 1 and Table 1 helps us to visualize them.

Now in order to better assess the performance, we present in Fig. 1 how the arrival rates  $\mu_0$  affect the effective throughputs  $\mathcal{R}_0^*$  for different values of  $\bar{p}$ . First we observe that when the same arrival rate is considered, the higher the  $\bar{p}$ , the lower the  $\mathcal{R}_0^*$ . This behavior is indeed intuitive since  $\bar{p}$  is related to the number of active links in the network and thus it determines the interference experienced by  $\text{RX}_0$ .

We also see that the effective throughput  $\mathcal{R}_0^*$  is a concave function of  $\mu_0$  (this statement can be easily proved from equation (17), considering  $0 < \mu_0 \leq 1$ ). For low rate of arrivals, the design setting used to reach  $\mathcal{R}_0^*$  should hold the packets more time in the queue in order to make  $\rho_0 \rightarrow 1$ . We also infer that low values of  $\mu_0$  also limit  $\mathcal{R}_0^*$  since the optimal design choice surprisingly yields a high outage probability in order to maintain the load of the queue close to one. Increasing  $\mu_0$ , on the other hand, the effect of this limitation diminishes and thus the effective throughput  $\mathcal{R}_0^*$  also increases until it reaches its maximum. After this inflection point, any increase of  $\mu_0$  degrades the link performance, indicating that high arrival rates are shrinking the feasible designing options, as discussed before.

Now, to study that maximum value, we present in Fig. 2 the effective throughput  $\mathcal{R}_0^*$  and its upper bound  $\mathcal{R}_{0,\text{up}}$  as a function of  $\mu_0$  when  $\bar{p} = 1$ . From the curves, we can verify that the maximum  $\mathcal{R}_0^*$  reaches its upper bound<sup>6</sup> given by (22), showing that, under certain circumstances, it is possible to obtain the unconstrained effective throughput  $\mathcal{R}_{0,\text{up}}$  even assuming strong requirements of queue stability and bounded packet loss probability.

In addition, we present in this figure together with the curves obtained using our analytical approximation the actual optimal values of  $\mathcal{R}_0^*$  which are computed using standard numerical procedures of Mathematica software, namely NMaximize or NMaxValue. As one can clearly verify,

6. Even though Fig. 2 only shows the results for  $\bar{p} = 1$ , the same analysis is still valid when other values of  $\bar{p}$  are considered.

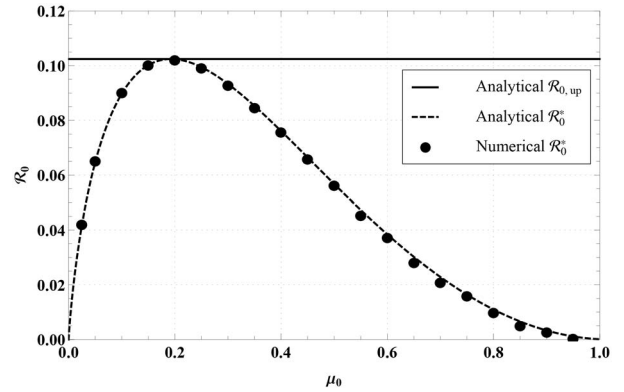


Fig. 2. Optimal effective link throughput  $\mathcal{R}_0^*$  of the typical link  $\text{TX}_0\text{--RX}_0$  and its upper bound  $\mathcal{R}_{0,\text{up}}$  (see Proposition 5) as a function of the arrival rate  $\mu_0$  considering  $\bar{p} = 1$ ,  $\alpha = 4$ ,  $d = 1$ ,  $\bar{p} \rightarrow 1$ ,  $\lambda_0 = 0.5$ , and  $\epsilon = 0.02$ . The optimal throughput  $\mathcal{R}_0^*$  is analytically assessed using Proposition 4 and via numerical optimization NMaximize or NMaxValue using Mathematica.

our approximation provides a good matching with the exact numerical solution<sup>7</sup>, evincing that the assumptions used to derive Proposition 4 are fairly reasonable. Nevertheless, it is worth pointing out that our approximation works properly for small values of  $\epsilon$ . When this condition is relaxed, the approximation  $P_{\ell,0}^* = P_{0,0}^{*1+m_0^*} \approx \epsilon$  does not hold anymore and consequently our result becomes weaker.

Figs. 1 and 2 also evince a necessary condition for achieving the effective throughput. From Theorem 1, it is possible to state that all effective throughputs above the  $\mathcal{R}_0^*$  curves cannot be achievable under the stability and packet loss constraints, determining therefore the boundary of stability for effective throughputs that the typical link  $\text{TX}_0\text{--RX}_0$  can achieve with bounded packet loss for a given network specification. In other words, if any effective throughput  $\mathcal{R}_0$  is stably achievable and the packet loss probability is at most  $\epsilon$ , then  $\mathcal{R}_0 < \mathcal{R}_0^*$  must hold.

#### 4 SPATIAL THROUGHPUT OPTIMIZATION IN POISSON NETWORKS

We analyze here the aggregate performance of the network using the spatial throughput metric introduced in Definition 3, i.e. stable achievable spatial throughput such that the packet loss probability is bounded for all links. We consider the Poisson random network described in the previous section and formulate an optimization problem in order to maximize the spatial throughput under queue stability and bounded packet loss probability for all links. Note that the infinite Poisson network model is equivalent in distribution to the limit of a sequence of finite networks with a fixed density as the area increases to infinity. Using similar steps to the proof of Proposition 4, we derive an approximated closed-form solution for such a problem, which allows us to compare the optimal spatial throughput to the spatial throughput byproduct of the optimal individual decisions given by Corollary 5.

7. To maintain the quality of the figures, we present the points obtained through numerical optimization only in some plots. In any case, the same good match is seen for other different network conditions.



### 4.1 Analytical Results

We consider the symmetric case where all  $\text{TX}_k\text{-RX}_k, \forall k \in \mathcal{N}_0$  are subject to the same arrival rates  $\mu$  and employ the same design parameters, namely access probability  $p$ , rate  $R$ , and maximum number of retransmissions of packets decoded in error  $m$ , resulting in the server rate  $\theta$ . Recalling that the set  $\mathcal{N}_0$  refers to all TXs generated by the homogeneous PPP  $\Phi_0$  with density  $\lambda_0$ , we can rewrite the spatial throughput definition given by equation (6) as

$$S = \lambda_0 (1 - P_\ell) p \rho \frac{R}{1 + \bar{m}}, \quad (27)$$

where  $\rho = \mu/\theta$ ,  $P_\ell$  is the packet loss probability and  $\bar{m}$  is the average number of retransmissions.

To define the constrained spatial throughput maximization problem, we focus again on the typical link  $\text{TX}_0\text{-RX}_0$ , which is representative of the performance of any link  $\text{TX}_k\text{-RX}_k, k \in \mathcal{N}_0$  due to properties of Palm distribution of PPP and Slivnyak's theorem [7]. The constraints given by Propositions 1, 2, and 3 are applied, noticing though that in the symmetric case  $\bar{\rho} = \rho_0 = \rho$  and  $\bar{p} = p_0 = p$  as well as  $\theta_0 = \theta$ ,  $\beta_0 = \beta$  and  $m_0 = m$ . Furthermore, recall that  $1 + \bar{m} = \frac{1 - P_0^{1+m}}{1 - P_0}$ , where  $P_0 = 1 - e^{-p \frac{\mu}{\theta} \lambda_0 \kappa d^2 \beta^{2/\alpha}}$ , and that  $P_\ell = P_0^{1+m}$ .

The spatial throughput optimization problem under stability and packet loss constraints is formulated as follows:

$$\begin{aligned} \max_{(p, \beta, m)} \quad & \lambda_0 p \frac{\mu}{\theta} \log_2(1 + \beta) e^{-p \frac{\mu}{\theta} \lambda_0 \kappa d^2 \beta^{2/\alpha}} \\ \text{s.t.} \quad & \left(1 - e^{-p \frac{\mu}{\theta} \lambda_0 \kappa d^2 \beta^{2/\alpha}}\right)^{1+m} \leq \epsilon, \\ & \theta = p \frac{e^{-p \frac{\mu}{\theta} \lambda_0 \kappa d^2 \beta^{2/\alpha}}}{1 - \left(1 - e^{-p \frac{\mu}{\theta} \lambda_0 \kappa d^2 \beta^{2/\alpha}}\right)^{1+m}}, \\ & \theta \geq p \left[ \sum_{i=1}^{1+m} \binom{1+m}{i} (-1)^{i+1} e^{-p \lambda_0 (i-1) \kappa d^2 \beta^{2/\alpha}} \right]^{-1} > \mu. \end{aligned}$$

The above problem seems more intricate than the one in Section 3 as the parameters  $\bar{\rho}$  and  $\bar{p}$  are now variable, creating higher interdependence between the design parameters and the network performance. In order to derive a closed-form approximate solution, we proceed in a similar way as before.

#### Proposition 6 (highest constrained spatial throughput).

Given that the network is in steady state and assuming that  $m$  is a non-negative real number, the highest spatial throughput  $S^*$  such that every  $\text{TX}_k, \forall k \in \mathcal{N}_0$ , has stable queue and the packet loss probability of every  $\text{TX}_k\text{-RX}_k$  link is bounded by  $\epsilon$ , which has a small value, can be approximated by

$$S^* \approx \begin{cases} \mu \lambda_0 (1 - \epsilon) \log_2 \left( 1 + \left( \frac{1}{\mu \lambda_0 (1 - \epsilon) \kappa d^2 e} \right)^{\alpha/2} \right) \\ \text{for } \mu(1 - \epsilon) \leq e^{-1} \\ \\ \mu \lambda_0 (1 - \epsilon) \log_2 \left( 1 + \left( \frac{-\log(\mu(1 - \epsilon))}{\lambda_0 \kappa d^2} \right)^{\alpha/2} \right) \\ \text{for } \mu(1 - \epsilon) > e^{-1}, \end{cases} \quad (28)$$

where the optimal design parameters  $(p^*, \beta^*, m^*)$  used to achieve the approximated  $S^*$  when  $\mu(1 - \epsilon) \leq e^{-1}$  are

$$\begin{aligned} p^* &= \mu(1 - \epsilon)e, \\ \beta^* &= \left( \frac{1}{\mu \lambda_0 (1 - \epsilon) \kappa d^2 e} \right)^{\alpha/2}, \\ m^* &= \frac{1}{\log_\epsilon(1 - e^{-1})} - 1, \end{aligned}$$

while  $(p^*, \beta^*, m^*)$  for  $\mu(1 - \epsilon) > e^{-1}$  is computed as

$$\begin{aligned} p^* &= 1, \\ \beta^* &= \left( \frac{-\log(\mu(1 - \epsilon))}{\lambda_0 \kappa d^2} \right)^{\alpha/2}, \\ m^* &= \frac{1}{\log_\epsilon(1 - \mu(1 - \epsilon))} - 1. \end{aligned}$$

**Proof.** We apply similar steps as for the proof of Proposition 4 recalling that  $*$  refers either to the optimal parameter choice or to a function of it. We first use the fact that the optimal solution is achieved when all TXs' probability (in steady state) that their backlog is empty tends to 0, i.e.  $\rho^* \rightarrow 1$  (stability region boundary). We consider that the packet loss constraint is low enough so that to achieve  $S^*$  the packet loss probability reaches its maximum value, i.e.  $P_\ell^* = P_0^{*1+m^*} = \epsilon$ . Thus,

$$\begin{aligned} \mu \approx \theta^* &= p^* \frac{1 - P_0^*}{1 - P_0^{*1+m^*}} = p^* \frac{1 - \epsilon^{\frac{1}{1+m^*}}}{1 - \epsilon} \Rightarrow \\ &\Rightarrow p^* \left(1 - \epsilon^{\frac{1}{1+m^*}}\right) \approx \mu(1 - \epsilon). \end{aligned} \quad (29)$$

Recalling that in the optimal configuration the following equality should hold  $1 - P_0^* = e^{-p^* \lambda_0 \kappa d^2 \beta^{*2/\alpha}}$ ,  $\beta^*$  can be computed as

$$\beta^* = \left( \frac{\log(p^*) - \log(\mu(1 - \epsilon))}{p^* \lambda_0 \kappa d^2} \right)^{\alpha/2}. \quad (30)$$

Manipulating the spatial throughput (27) based on the arguments stated above yields

$$\begin{aligned} S^* &= \lambda_0 (1 - P_\ell^*) p^* \rho^* \frac{R^*}{1 + \bar{m}^*} \\ &= \lambda_0 p^* R^* (1 - P_0^*) \\ &\approx \mu \lambda_0 (1 - \epsilon) \times \\ &\quad \times \log_2 \left( 1 + \left( \frac{\log(p^*) - \log(\mu(1 - \epsilon))}{p^* \lambda_0 \kappa d^2} \right)^{\alpha/2} \right), \end{aligned} \quad (31)$$

where  $R^* = \log_2(1 + \beta^*)$ .

Note that the only design parameter in (31) is  $p^*$ , which can be thus computed as

$$\begin{aligned} p^* &= \arg \max_x \frac{\log(x) - \log(\mu(1 - \epsilon))}{x \lambda_0 \kappa d^2} \\ &= \min(\mu(1 - \epsilon)e, 1), \end{aligned} \quad (32)$$

where  $\mu < x \leq 1$ .

Placing (32) in (31) results in (28), and in order to obtain the optimal design parameters, we apply (32) into (30) so as to obtain  $\beta^*$  and then manipulate (29) to find  $m^*$ .  $\square$

The solution to the optimization problem stated above provides the highest achievable spatial throughput constrained by queue stability and bounded packet loss requirements for all links. Similarly to the previous section, the following corollaries follow from properties of the above result.

**Corollary 6.** *The spatial throughput  $S^*$  and the system parameters  $(\rho_0^*, \beta_0^*, m_0^*)$  stated in Proposition 6 have the following properties:*

- (a1)  $S^*$  is a concave function of  $\mu \in [0, 1]$  and  $\lambda_0 > 0$ ;
- (a2) The arrival rate  $\mu^* \in [0, 1]$  that maximizes  $S^*$  can be found as the  $\mu \in [0, 1]$  that is solution to the derivative equation:  $dS^*/d\mu = 0$ ;
- (a3) The network density  $\lambda_0 > 0$  that maximizes  $S^*$  can be found as the  $\lambda_0 > 0$  that is solution to the derivative equation:  $dS^*/d\lambda_0 = 0$ ;
- (b)  $p^*$  is a monotonically increasing function of  $\mu$  and is not affected by  $\lambda_0 > 0$ ;
- (c)  $\beta^*$  is a monotonically decreasing function of  $\mu \in [0, 1]$  and  $\lambda_0 > 0$ .
- (d)  $m^*$  is a monotonically decreasing function of  $\mu \in [0, 1]$  and  $\lambda_0 > 0$ .

The proof of this corollary comes directly from the analysis of the function stated in Proposition 6 and therefore it is not presented here.

**Proposition 7 (upper bound of  $S^*$ ).** *An upper bound on the highest spatial throughput  $S^*$  given by Proposition 6  $S_{\text{up}}$  is given by*

$$S^* \leq S_{\text{up}} = \begin{cases} \frac{\lambda_0 \log_2(1 + \beta^*)}{e^{\lambda_0 \kappa d^2 \beta^{*2/\alpha}}} & \text{for } \beta^* \leq (\lambda_0 \kappa d^2)^{-\alpha/2} \\ \frac{\log_2(1 + \beta^*)}{\kappa d^2 \beta^{*2/\alpha} e} & \text{for } \beta^* > (\lambda_0 \kappa d^2)^{-\alpha/2}, \end{cases} \quad (33)$$

where  $\beta^*$  is found as the value of  $\beta$  that is the solution to

$$\beta = \begin{cases} \left( \frac{2}{\alpha} \lambda_0 \kappa d^2 (1 + \beta) \log(1 + \beta) \right)^{\frac{\alpha}{\alpha-2}} & \text{for } \beta \leq (\lambda_0 \kappa d^2)^{-\alpha/2} \\ -1 + e^{\mathcal{W}_0(-\frac{\alpha}{2} e^{-\alpha/2}) + \frac{\alpha}{2}} & \text{for } \beta > (\lambda_0 \kappa d^2)^{-\alpha/2}, \end{cases} \quad (34)$$

where  $\mathcal{W}_0(\cdot)$  is the principal branch of the Lambert W function defined as  $x = \mathcal{W}_0(x)e^{\mathcal{W}_0(x)}$  such that  $x \geq -e^{-1}$  and  $\mathcal{W}_0(x) \geq -1$ .

**Proof.** To solve the unconstrained optimization, we first assume that the dominant network in which TXs have always packets to transmit (i.e.  $\rho \rightarrow 1$ ). We then compute the values of the access probability  $p^*$  and the SIR threshold  $\beta^*$  that lead to a feasible equilibrium point, noting that the spatial throughput given by  $S = \lambda_0 p \log_2(1 + \beta) e^{-p \lambda_0 \kappa d^2 \beta^{2/\alpha}}$  is a concave function of both  $\beta$  and  $p$ . From the derivative equations, we find the relation between  $p^*$  and  $\beta^*$  at the equilibrium point as

$$p^* = \frac{1}{\lambda_0 \kappa d^2 \beta^{*2/\alpha}}. \quad (35)$$

Note that since  $0 \leq p \leq 1$  some equilibrium points given by (35) may not lie in the feasibility region of the problem, thus whenever  $p^* > 1$ , we set  $p^* = 1$  and manipulate the equations accordingly. After some algebraic manipulations using (35), we obtain (33) and (34), concluding the proof.  $\square$

**Theorem 2 (necessary condition for spatial throughput).**

*Given that the network is in steady state, a necessary condition so that any spatial throughput  $S$  is achievable with bounded packet loss probability and queue stability for all TXs is*

$$S < S^*, \quad (36)$$

where  $S^*$  is given by Proposition 6.

The proof of Theorem 2 is similar to the proof of Theorem 1 from the previous section and thus it is omitted. Next we provide two important corollaries identifying how  $S^*$  is related to the spatial throughput  $S_{\text{ind}}^*$  reached when all links use the best individual design parameters stated in Corollary 5.

**Corollary 7 (network optimal vs. per-link optimal).** *Given that the network is in steady state, then in the symmetric case*

$$S_{\text{ind}}^* \leq S^*, \quad (37)$$

where  $S^*$  is given by Proposition 6 and  $S_{\text{ind}}^*$  by Corollary 5. Equality in (37) occurs whenever  $p^* = 1$ .

**Corollary 8 (Tragedy of the commons).** *Given that the network is in steady state, then in the symmetric case the best individual design setting derived in Section 3 is not globally optimal for the aggregate network performance when  $\mu(1 - \epsilon) \leq e^{-1}$ . In other words, the selfish behavior of the TXs leads to poor network resource utilization, degrading its spatial throughput. This degradation in the aggregate performance due to selfish per-link decisions can be seen as a tragedy of the commons class of problem [28].*

**Remark.** The ratio between  $S_{\text{ind}}^*$  and  $S^*$  can be directly obtained from equations (26) and (28). From such ratio, one can verify that  $S_{\text{ind}}^* = S^*$  when  $\mu(1 - \epsilon) > e^{-1}$ , exactly when the access probability  $p^*$  that leads to  $S^*$  becomes 1. It is also interesting to note that the range of arrival rates where selfish decisions are not optimal is  $\mu(1 - \epsilon) \leq e^{-1}$ . As our basic assumption is that the maximum acceptable packet loss probability  $\epsilon$  is small, one can see that  $\mu$  determines the feasibility region of our optimization regardless of the design setting used to achieve the optimal performance. In other words, by increasing  $\mu$ , the probability  $p^*$  that is the spatial rate  $S^*$  optimizer monotonically increases up to  $p^* = 1$ , independently of other system variables (c.f. Corollary 6). Once  $p^*$  is determined, the other parameters must be optimally tuned in accordance with the system constraints.

The proofs of these corollaries are straightforward and thus they are omitted.

## 4.2 Numerical Results

In this section, we provide numerical results in order to verify the aforementioned analytical results. In Table 2 we

8. Note that the computation of  $S_{\text{up}}$  involves a simple numerical procedure to solve the first equation in (34).

TABLE 2  
Optimal Spatial Throughput Design Setting for  $\alpha = 4$ ,  $d = 1$ ,  
and  $\epsilon = 0.02$

$(\lambda_0, \mu)$	$(p^*, R^*, 1 + m^*)$	$S_{\text{ind}}^*$	$S^*$	$S_{\text{up}}$
(0.1, 0.2)	(0.53, 3, 95, 8.5)	0.070	0.077	0.0865
(0.1, 0.8)	(1, 0.31, 2.6)	0.025	0.025	0.0865
(0.5, 0.2)	(0.51, 0.66, 8.5)	0.051	0.065	0.0865
(0.5, 0.8)	(1, 0.014, 2.6)	0.005	0.005	0.0865

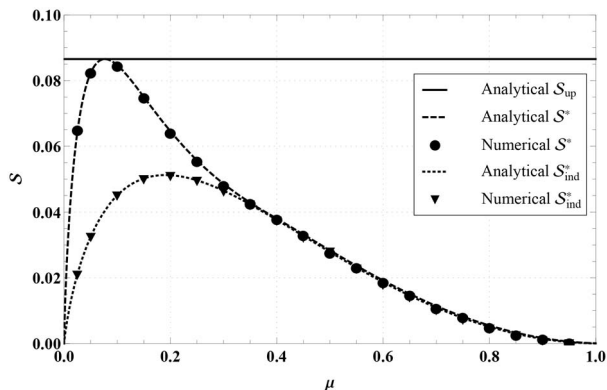


Fig. 3. Optimal network spatial throughput  $S^*$ , its upper bound  $S_{\text{up}}$  (see Proposition 7) and the spatial throughput  $S_{\text{ind}}^*$  obtained with the best individual choice as a function of the arrival rate  $\mu$  for  $\lambda_0 = 0.5$ ,  $\alpha = 4$ ,  $d = 1$  and  $\epsilon = 0.02$ . The spatial throughputs  $S^*$  and  $S_{\text{ind}}^*$  are analytically assessed using Proposition 5 and Corollary 5, respectively, and they are also computed via numerical optimization NMaximize or NMaxValue from Wolfram Mathematica package.

present the design setting  $(p^*, \beta^*, m^*)$  that leads to the highest spatial throughput achieved when stability and packet loss constraints are required for all links, considering different combinations of the input parameters  $\lambda_0$  and  $\mu$ .

From Table 2, it is verified that in scenarios with low values of  $\mu$ , e.g.  $\mu = 0.2$ , the access probability  $p^*$  is about 0.5, whereas when  $\mu = 0.8$ , it approaches the value of the link optimization case, i.e.  $p^* = 1$ . These facts indicate that, when the network is not limited by high arrival rates, it is important to have some kind of medium access control so that the effects of co-channel interference are weakened. On the other hand, increasing the arrival rates, the stability constraint makes the optimal access probability  $p^*$  become higher, reaching 1. This reflects that, in scenarios where the queue stability restriction is the dominant factor, the optimal link decisions are also optimal from a network point of view, i.e.  $S^* = S_{\text{ind}}^*$ . In any case, when  $p^* = 1$ ,  $S^*$  is remarkably lower than its upper bound  $S_{\text{up}}$  given in Proposition 7.

Fig. 3 shows the spatial throughput  $S^*$  together with  $S_{\text{up}}$  and  $S_{\text{ind}}^*$  versus the arrival rate<sup>9</sup>. As argued before, we deduce that for lower values of  $\mu$ , the performance gap between the spatial throughputs  $S^*$  and  $S_{\text{ind}}^*$  is big, reaching 100% for some values of  $\mu$ . This gap closes for increasing arrival rates  $\mu$ . More interestingly, we can see

9. Once again we can see very good matching between the proposed approximation and the results obtained via numerical optimization.

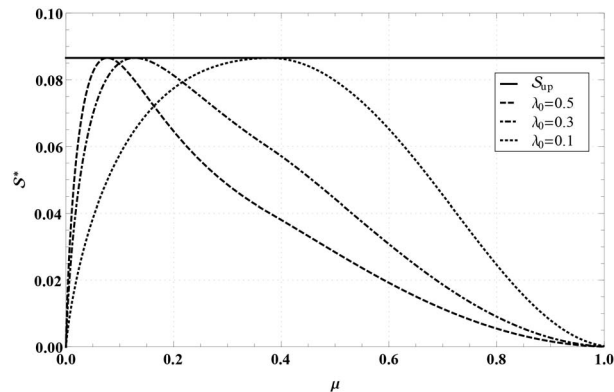


Fig. 4. Optimal spatial throughput  $S^*$  (see Proposition 6) and its upper bound  $S_{\text{up}}$  (see Proposition 7) versus the arrival rate  $\mu$  for different densities  $\lambda_0$ , considering  $\alpha = 4$ ,  $d = 1$ , and  $\epsilon = 0.02$ .

from Fig. 3 that the constrained spatial throughput  $S^*$  can achieve values very close to its upper bound given by the unconstrained spatial throughput optimization.

As discussed in Section 3.3, to reach the optimal performance under queue stability constraint, all TXs have to transmit with high probability when the arrival rate increases. When the unconstrained optimization problem is considered, however, the opposite happens: the optimal performance is achievable by *decreasing* the access probability, thus controlling the interference level of the network by contention (see equation (35)). In other words, increasing the arrival rates  $\mu$ , the stability constraint makes the access probability be far away from its optimal unconstrained value. Nevertheless,  $S^*$  can still reach the unconstrained spatial throughput for some specific combinations of  $\mu$  and  $\lambda_0$ , as shown by Figs. 4 and 5.

Fig. 4 presents how  $S^*$  behaves as a function of the arrival rates  $\mu$  for different values of density  $\lambda_0$ . For low values of  $\mu$ ,  $S^*$  increases as  $\mu$  increases until it reaches  $S_{\text{up}}$ . At lower densities,  $\lambda_0$  can be viewed as the limiting factor of  $S^*$  due to low spatial reuse, therefore such an inflection point is reached at higher arrival rates  $\mu$  for lower densities  $\lambda_0$ . After its maximum value,  $S^*$  decreases as  $\mu$  increases, approaching zero when  $\mu$  goes to 1, regardless of the density considered. This once again corroborates

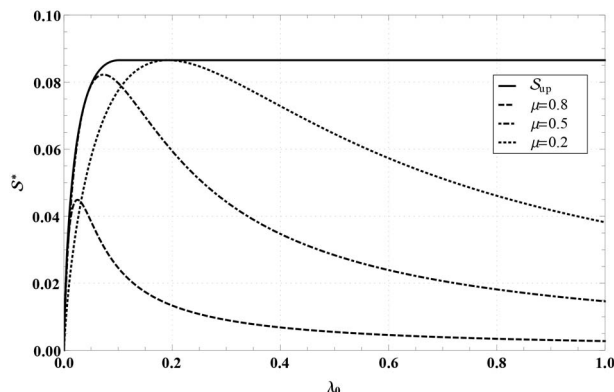


Fig. 5. Optimal spatial throughput  $S^*$  (see Proposition 6) and its upper bound  $S_{\text{up}}$  (see Proposition 7) versus the network density  $\lambda_0$  for different arrival rates  $\mu$ , considering  $\alpha = 4$ ,  $d = 1$  and  $\epsilon = 0.02$ .



the intuition that high arrival rates degrade the network efficiency.

In Fig. 5 we see that increasing the node density  $\lambda_0$ , the values of  $S^*$  and  $S_{\text{up}}$  increase up to a maximum, which indicates that the network is limited by the low spatial density of TXs. Conversely, once such maximum point is reached, which is at lower densities  $\lambda_0$  for higher arrival rates  $\mu$  (justified by the same arguments used before),  $S^*$  becomes a decreasing function of  $\lambda_0$  while  $S_{\text{up}}$  is able to maintain its best performance regardless of  $\lambda_0$  due to contention, reflecting that the interference from the concurrent transmissions starts dominating the network performance when the constraints are imposed.

Interestingly, for lower values of  $\lambda_0$ ,  $S^*$  is very close to its upper bound  $S_{\text{up}}$  when arrival rates  $\mu = 0.5$  and  $\mu = 0.7$  are considered, while the network has poorer performance for  $\mu = 0.2$ . These facts indicate that a sparse network subject to low traffic conditions operates below its limit, which can be achieved when the arrival rates are increased. On the other hand, when denser network scenarios are considered,  $\mu = 0.2$  leads to higher  $S^*$  than the other arrival rates studied here. All in all, these facts reinforce our argument that, under certain conditions, it is possible to achieve the unconstrained performance via a suitable parameter design, even though strong requirements in terms of packet loss and stability are imposed.

## 5 CONCLUSION

In this paper, we investigated the performance of random spatial networks in terms of effective link throughput and the network-wide spatial throughput under a queue stability constraint and bounded packet loss probability. Considering an ad hoc network in which transmit nodes are located according to a homogeneous Poisson point process and are subjected to geometric packet arrivals, we showed under which conditions it is possible to achieve the non-constrained throughput performance and also established a necessary condition so that both throughputs are achievable under the above constraints. Furthermore, we proved that the individual link design parameters that lead to the highest effective link throughput are not always a wise choice for maximizing the network spatial throughput, identifying when the solutions of both optimization problems coincide.

Future work may include applying the insights provided here to design distributed algorithms based on locally estimated network parameters in more realistic environments, such as multi-hop ad hoc networks. Finally, clustered local decisions and different mobility models introducing spatio-temporal correlation between events have to be considered.

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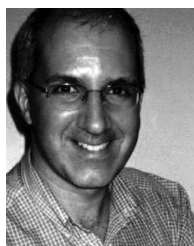


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