

Effects of Mobility on User Energy Consumption and Total Throughput in a Massive MIMO System

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Abstract—Macroscopic mobility of users is important to determine the performance and energy efficiency of a wireless network, because of the temporal correlations it introduces in the consumed power and throughput. In this work, we introduce a methodology that allows to compute the long time statistics of such metrics in a network. After describing the general approach, we consider a specific example of the uplink channel of a mobile user in the vicinity of a base station equipped with a large number of antennas (the so called "massive MIMO" base station). To guarantee a fixed signal-to-noise ratio and rate, the user inverts the pathloss channel power, while moving around in the cell. To calculate the long time distribution of the corresponding consumed energy, we assume that its movement follows a Brownian motion, and then map the problem to the solution of the minimum eigenvalue of a partial differential equation, which can be solved either analytically, or numerically very fast. The single-user throughput is also treated. We then present some results and discuss how they can be generalized if the mobility model is assumed to be a Levy random walk. A roadmap to use this methodology is eventually given to extend results to a multiple user set-up with multiple base stations.

I. INTRODUCTION

One of the most challenging properties of the wireless propagation channel is its temporal variability. Since the first mobile telephones appeared at a massive scale, engineers had to address the temporal fluctuations of the received signal power to make sure that a call connection remained active. Adverse effects that needed to be countered include: (i) fading holes of the channel due to multiple wave reflections; (ii) change in the link distance due to physical movement of the wireless device away from the base; or (iii) interference fluctuations due to movement of interfering devices.

To understand the behavior of fading various models were proposed with varying complexity, starting from the Jakes model [1] to more involved correlated models in both frequency and time [2]. The introduction of concise fading models made it possible to obtain, together with numerical simulations, analytical expressions for the quantities of interest, such as ergodic and outage capacities [3]. This in turn allowed the introduction of ways to counter the adverse effects of fading, through scheduling and space-time coding.

The effects of pathloss have also been studied more recently in a large body of work using the theory of Poisson point processes (PPP) [4]–[6]. This approach has provided a good

understanding of the effects of randomness in position of mobile devices in a network and has allowed a thorough characterization of the statistics of interference in wireless networks. However, most of the analysis deals with static or near-static networks and does not take into account the consequences of macroscopic mobility of users.

This shortcoming is important when one realizes that due to not too rapid mobility there are temporal correlations in the necessary power for any given user. For example, an untypically high density of users at the cell edge will result to an increased energy consumption over an extended period of time, which may drain the available energy resources of a base station (BS). This, in turn, becomes important especially for off-grid deployments, with finite energy resources. Therefore, it is of paramount importance to quantify not only how often such unlikely events happen, but also how long they last, which depends on the user mobility.

To analyze the effects of mobility, simple yet effective models that describe the statistics of humans moving around are necessary. Several models have been proposed [7], [8] and their statistical properties have been studied [9] in detail and have been implemented in numerical simulators. In particular, three types of mobility models are more popular. The first and simplest one is a random walk (RW). This model is a continuous time Markov process on a lattice with a step size distribution, which has zero mean and finite variance. At long times and distances this can be approximated by a Brownian motion (BM). Another more involved model is a Markov process (LRW) with infinite variance in the step size distribution, which corresponding to Levy processes (due to the long tails in the step sizes). This has been proposed as a more realistic model for human mobility [10], [11]. Finally, the so-called random waypoint process (RWP) has also been proposed, in which a mobile user picks a random destination and travels with constant velocity to reach it, which is also a Markov process. However, despite the well-understood properties of the mobility models above not much progress [12] has been made towards providing analytical results for the long term statistics of communications performance metrics, such as energy consumed or total throughput.

In this work, we take advantage of the Markovian property of user mobility to analyze the long time statistics of these performance metrics in a network with mobility. We believe that the approach is fairly general to encompass (at least in principle) all Markovian mobility models described above. The

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methodology is based on a simple, but powerful theorem, the so-called Feynman-Kac formula [13], [14], which maps the average over all random walks to the minimum eigenvalue of a partial differential equation that is related to the random walk.

Next section defines the metrics of interest (in terms of an integral over time), introduces the mobility model and show how the probability of finding a user at a given location can be computed using the diffusion equation. In Section III, the main result of the paper is given, namely, Theorem 1 and outline its proof. In Section IV, we calculate the long time statistics of the energy consumption in the uplink of a single-user massive MIMO system, while the long time statistics of the corresponding throughput are computed in Section V. Finally, in Section VI we discuss how the proposed methodology can be generalized to other scenarios.

II. PROBLEM STATEMENT AND SETUP

The purpose of this paper is to present a methodology to analyze the long time T statistics of quantities of the form

$$E_T = \int_0^T dt V(\mathbf{r}(t)) \quad (1)$$

where $V(\mathbf{x})$ is a function of the position of one or more mobile devices in a network and the devices move around the region of interest over time. In all subsequent discussions, we assume that $V(\mathbf{x})$ is greater than or equal to zero and it is bounded from above.

A. Metrics of Interest

$V(\mathbf{x})$ is a function that can represent a number of relevant metrics of interest. In this work, we limit to consider the two specific functions defined below.

1) *Consumed energy*: In this case, $V(\mathbf{x})$ is proportional to the inverse of the pathloss function between a given user and the nearest BS. Hence, we have

$$V_p(\mathbf{r}) = P(\mathbf{r}) = \gamma r^\beta \quad (2)$$

where β is the pathloss exponent and r is the distance between the user and the BS. Here, the integral in (1) will correspond to the total energy consumption of the mobile over time T . For simplicity, we treat only a single cell which we take to be a square with side R and the BS located at the centre. This problem may be generalized straightforwardly to a network of many BSs located for example at a square grid. In this case, the above distance r would be replaced by $r_{min} = \min_{\mathbf{r}_i} |\mathbf{r} - \mathbf{r}_i|^\beta$ where \mathbf{r}_i is the location of the i th BS.

The quantity in the right-hand-side of (2) is nothing else than the power required by a user at a distance r from the BS to maintain a signal-to-noise ratio (SNR) equal to γ in the presence of unit variance noise, no other interference and no fading. Despite its simplicity, it is not difficult to show that this is asymptotically correct in the uplink of a massive MIMO system, where a finite number of users K is served with a very

large antenna array of N antennas at the BS. Indeed, the N dimensional received signal vector \mathbf{y} at the BS is given by

$$\mathbf{y} = \sum_{k=1}^K \mathbf{h}_k g(\mathbf{r}_k)^{1/2} x_k + \mathbf{z} \quad (3)$$

where \mathbf{h}_k for $k = 1, \dots, K$ is the N -dimensional channel vector for user k , with elements assumed for simplicity to be $\sim \mathcal{CN}(0, N^{-1})$, $g(\mathbf{r}) = |\mathbf{r}|^{-\beta}$ is the corresponding pathloss function and x_k the transmitted signal, with \mathbf{z} being the N -dimensional noise vector with elements $\sim \mathcal{CN}(0, 1)$. Then, in the limit $N \gg K$ the SINR for each user reduces to [3]

$$\text{SNR}_k = g(\mathbf{r}_k) \mathbb{E} [|x_k|^2]. \quad (4)$$

Setting the right-hand-side of the above equation equal to γ leads to (2). The pathloss function inversion can easily be implemented through the periodic feedback of a channel quality indication (CQI) to the mobile device. Hence, the above power control scheme corresponds to situations where the user needs a constant rate.

2) *Throughput*: A dual uplink transmission strategy to the above for a mobile user in a network corresponds to transmit continuously at a constant power and take advantage of the instances when the channel is good due to proximity to a BS. In this case the power transmitted is fixed, but it is the communication rate that is fluctuating with distance. This can be expressed as

$$V_c(\mathbf{r}) = C(\mathbf{r}) = \alpha \min \left(\log \left[1 + \frac{p}{r^\beta} \right], R_{max} \right) \quad (5)$$

where r is defined as above, $R_{max} = \log(1 + p/r_0^\beta)$ is the maximum rate achieved at distance r_0 and α, p are constant parameters. Here, the integral of (1) will correspond to the total throughput uploaded over time t in bits. The above expression has two interpretations, depending on the context. In the case of a single BS serving a single user, it corresponds to the outage capacity at location r_{min} . In this case, $\alpha = 1 - \mathbb{P}_{out}$ is the probability of non-outage, while $p = -P \log(1 - \mathbb{P}_{out})$ and $\int dt C(\mathbf{r}(t))$ will correspond to the total goodput. In a massive MIMO multi-user setting, $\alpha = 1$ and p is the SNR.

B. Mobility Model

The dynamics of user mobility are now specified. In particular, we assume that the user of interest moves according to a continuous time Markov process. The infinitesimal generator of the process is denoted by the operator \mathbf{M}_0 acting on the space of square integrable functions $\ell^2(\mathbb{R}^2)$. Hence, the probability that a user is at location \mathbf{r} at time t , given that it was at location \mathbf{r}_0 at time $t = 0$ can be expressed in terms of \mathbf{M}_0 as follows [13]

$$\mathbb{P}(\mathbf{r}, t; \mathbf{r}_0, 0) = e^{-\mathbf{M}_0 t}(\mathbf{r}, \mathbf{r}_0) \quad (6)$$

where the right-hand-side is the $(\mathbf{r}, \mathbf{r}_0)$ -th element of the exponential operator. For concreteness, we only assume the user performs the simplest Markov process, namely, a Brownian motion. This is known to be a good approximation for the long time and large distance properties of a Markov process

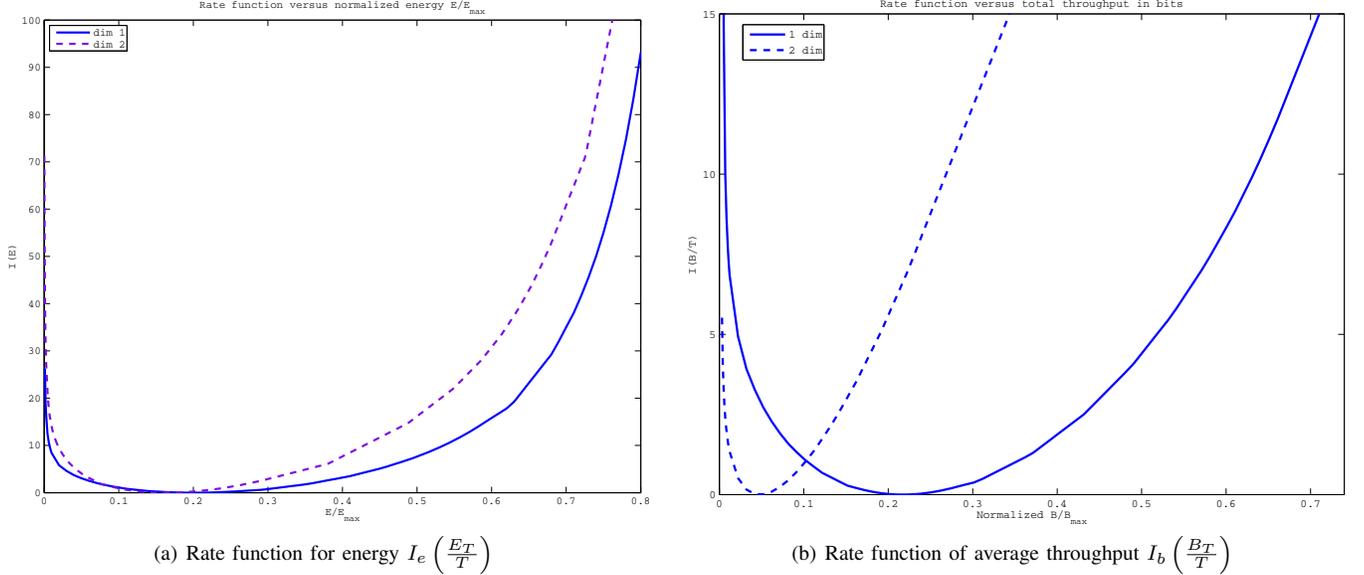


Fig. 1. In Fig. 1(a) we plot the rate function for the distribution of the energy of the mobile user. The rate function is plotted for both a one- and two-dimensional square cell. To fit both curves in the same plot the x -axis indicates the energy value normalized to its maximal value. Although close to the point where the rate function vanishes, which occurs at the mean value of the energy, the shape is quadratic, its distinctly non-Gaussian shape becomes evident beyond that region. The diverging behavior at the two boundary values, as analyzed in Section IV are visible in the plot. In Fig. 1(b) the rate function for the throughput distribution is plotted. The values $\beta = 4$, $p = 0.01$, $r_0 = 0.1$ and $\alpha = 1$ are used. The very different behavior between one- and two-dimensional rate values for large values of B are due to the lower density of points for $r < r_0$ for two- versus one-dimensional cells.

with finite step variance [15]. In this case, the infinitesimal generator is simply given by

$$\mathbf{M}_0 = -\frac{D}{2}\nabla^2 \quad (7)$$

where ∇ is the Laplacian operator and D is the diffusion constant. In this case, the probability in (6) satisfies the diffusion equation

$$\frac{\partial \mathbb{P}(\mathbf{r}, t; \mathbf{r}_0, 0)}{\partial t} = \frac{D}{2}\nabla^2 \mathbb{P}(\mathbf{r}, t; \mathbf{r}_0, 0) \quad (8)$$

with initial condition $\mathbb{P}(\mathbf{r}, 0; \mathbf{r}_0, 0) = \delta(\mathbf{r} - \mathbf{r}_0)$, where $\delta(\mathbf{x})$ is the two dimensional Dirac δ -function. We also need to specify the boundary conditions of the Brownian motion. Specifically, we assume periodic boundary conditions. This means that the mobile user, when it moves through the boundary of the cell, re-appears from the other side with the same direction. While this is not particularly realistic for a user, it is not hard to see that this is a good way to mimic the hand-over to a neighbouring cell region. This fact will be discussed further later on.

III. MATHEMATICAL FRAMEWORK

In this section, the main result of this work is given, which provides the long time behaviour of the metrics introduced above. Then, the limiting behaviour for small and large values of both energy and throughput as well as the behaviour close to the mean will be analytically characterized.

Theorem 1. *Let $V(\mathbf{r}) \geq 0$ be a continuous, upper bounded function of the distance \mathbf{r} and E_T be given by (1), in which the time-dependence of the position $\mathbf{r}(t)$ is due to a Brownian*

motion on a square $\mathbb{B} = (-R/2, R/2)^2$ with diffusion constant D , periodic boundary conditions and initial condition $\mathbf{r}_0 = \mathbf{r}(0)$. Then if $A \subset \mathbb{R}$ we have

$$\lim_{T \rightarrow \infty} \frac{1}{T} \log \mathbb{P}(E_T/T \in A) = -\inf_{x \in A} I(x) \quad (9)$$

$$I(x) = -\inf_{\lambda \in \mathbb{R}} \{\lambda x - \epsilon_0(\lambda)\} \quad (10)$$

where $\epsilon_0(\lambda)$ is the minimum eigenvalue of the operator

$$\mathbf{M}_\lambda = -\frac{D}{2}\nabla^2 + \lambda V(\mathbf{r}). \quad (11)$$

Proof. We now sketch the basic steps of the proof. We start by applying Cramer's theorem [16] to find that in the limit of large T the quantity $\log \mathbb{P}(E_T \in A)$ obeys a large deviation principle with rate function $I(x)$ so that

$$\lim_{T \rightarrow \infty} \frac{1}{T} \log \mathbb{P}(E_T/T \in A) = -\inf_{x \in A} I(x) \quad (12)$$

$$I(x) = -\inf_{\lambda \in \mathbb{R}} \{\lambda x + \Lambda(\lambda)\} \quad (13)$$

$$\Lambda(\lambda) = \lim_{T \rightarrow \infty} \frac{1}{T} \log \mathbb{E}_{\mathbf{r}_0} [e^{-\lambda E_T}]. \quad (14)$$

In the above, the expectation is computed over Brownian paths (random walks) with initial condition $\mathbf{r}(0) = \mathbf{r}_0$. Also, note that the second line has an inf rather than a sup as it is customary since we have $\Lambda(\lambda)$ with a negative sign in the exponent.

Now, the main trick in the proof is to take advantage of a famous result, namely, the Feynman-Kac (FK) formula, which states that [13], [14]

$$e^{-\mathbf{M}_\lambda T}(\mathbf{r}_0, \mathbf{r}_T) = \mathbb{P}(\mathbf{r}_T, T; \mathbf{r}_0, 0) \mathbb{E}_{\mathbf{r}_0, \mathbf{r}_T} [e^{-\lambda E_T}] \quad (15)$$

where the right-hand-side is an expectation over all Brownian motions starting at r_0 and ending at r_T after a time interval T . The operator \mathbf{M}_0 is the infinitesimal generator of the semigroup corresponding to the mobility process and it is given by (7) when the simple Brownian motion is considered. To relate the above equation to the right-hand-side of (14), we need to integrate the above result over \mathbf{r}_T with the appropriate probability of the path given by $\mathbb{P}(\mathbf{r}_T, T; \mathbf{r}_0, 0)$.

The left-hand side of the FK formula can be expressed very simply using the spectral decomposition of \mathbf{M}_λ . Let $\phi_n(\mathbf{r})$ be the eigenfunctions of \mathbf{M}_λ with corresponding eigenvalue $\epsilon_n(\lambda)$. Then, we obtain

$$e^{-\mathbf{M}_\lambda T}(\mathbf{r}_0, \mathbf{r}) = \sum_{n=0}^{\infty} \phi_n(\mathbf{r}_0) \phi_n(\mathbf{r}) e^{-\epsilon_n(\lambda)T}. \quad (16)$$

The periodic boundary conditions imposed above mean that the eigenfunctions $\phi_n(\mathbf{r})$ and their derivatives $\nabla \phi_n(\mathbf{r})$ have to be continuous on opposite boundaries, i.e., $(x, -R/2) \rightarrow (x, R/2)$ and $(-R/2, y) \rightarrow (R/2, y)$. Integrating over the final position \mathbf{r}_T yields

$$\begin{aligned} \mathbb{E}_{\mathbf{r}_0} [e^{-\lambda E_T}] &= \sum_{n=0}^{\infty} \theta_n \phi_n(\mathbf{r}_0) e^{-\epsilon_n(\lambda)T} \quad (17) \\ \theta_n &= \int_{\mathbf{r} \in \mathbb{B}} \phi_n(\mathbf{r}) d\mathbf{r}. \end{aligned}$$

As a result, in the large T limit one gets

$$\mathbb{E}_{\mathbf{r}_0} [e^{-\lambda E_T}] \approx \theta_0 \phi_0(\mathbf{r}_0) e^{-\epsilon_0(\lambda)T}. \quad (18)$$

Combining the above result with (13) and (14) completes the proof. \square

Remark. It should be pointed out that there are analogous (but non-local) expressions for generators of stable processes as well [17]. Also, discrete analogues of the Laplacian can also be found, corresponding to discrete space (continuous time) Markov processes. The FK formula essentially finds the right way to weight the dynamics of the user for which all locations in the cell are equal and the weighting of V , which is different as a function of \mathbf{r} .

As seen, in the large T limit we only need to find the minimum eigenvalue $\epsilon_0(\lambda)$ of the operator \mathbf{M}_λ for all λ . The latter must be then plugged into (13) from which λ is eventually computed. Both steps can be done numerically with a reasonable effort. However, limiting results for the tails of the distribution can also be computed analytically as shown next.

IV. ENERGY STATISTICS

In this section, we analyze the energy statistics, computing

$$E_T = \gamma \int_0^T r(t)^\beta dt. \quad (19)$$

Plugging this into the methodology above we obtain the rate function $I_e(x)$, which provides the leading (exponential) term in the distribution of E_T for large T . We first find the

minimum eigenvalue of the operator in (11) for $V(\mathbf{r}) = \gamma r^\beta$. We initially derive limiting results for large and small values of E_T . We start with the case of large positive λ . This corresponds to the probability tails for small energy values. In this case, the minimum eigenvalue has an eigenfunction localized close to the centre of the cell. As a first approximation, we can neglect the cell boundary. Therefore, the distance variable can be rescaled to $\mathbf{r} = \bar{r}\mathbf{x}$, where $\bar{r} = (D\lambda^{-1}\gamma^{-1})^{1/(\beta+2)}$ eliminating the dependence on λ . The resulting minimum eigenvalue becomes approximately equal to

$$\epsilon(\lambda) \approx D^{\frac{\beta}{\beta+2}} \gamma^{\frac{2}{\beta+2}} \lambda^{\frac{2}{\beta+2}} \epsilon_0 \quad (20)$$

where ϵ_0 is the minimum energy of \mathbf{M}_λ in \mathbb{R}^2 with $\lambda = D = 1$. After some algebra, we obtain for $E_T \ll \gamma R^\beta T$

$$I_e(x) \approx \frac{D}{R^2} \left\{ \frac{\beta}{2} \left(\frac{2\epsilon_0}{\beta+2} \right)^{1+\frac{2}{\beta}} \right\} \left(\frac{x}{\gamma R^\beta} \right)^{-\frac{2}{\beta}} \quad (21)$$

To obtain the tails for $E_T \gg P_{mean}T$, we analyze the case for large negative values of λ . We observe that the minimum of $\lambda V(\mathbf{r})$ is achieved at the cell corners. This means that the term $|\mathbf{r}|^\beta$ must be expanded around the value r_{max}^β , $r_{max} = R/\sqrt{2}$. After a shift and 45° rotation of axes, we obtain

$$\mathbf{M}_\lambda \approx \lambda P_{max} + \mathbf{M}_0 - \lambda \gamma \beta r_{max}^{\beta-1} \max(|x|, |y|) + O(|\lambda| r_{max}^{\beta-2}) \quad (22)$$

where $P_{max} = \gamma r_{max}^\beta$. In this case, one gets

$$I_e(x) \approx \frac{D}{R^2} \frac{4\epsilon_m^3}{27} \left(1 - \frac{x}{P_{max}} \right)^{-2} \quad (23)$$

where ϵ_m is the minimum eigenvalue of the ‘‘inverted tetrahedron’’ $L^2(\mathbb{R}^2)$ operator

$$\mathbf{M}_{eff} = -\frac{1}{2} \nabla^2 + \max(|x|, |y|) \quad (24)$$

Finally, we may obtain the behavior for $E_T \approx TP_{mean}$. With some hindsight, we look in the region of small $|x|$. In this case, $\lambda V(\mathbf{r})$ is assumed to be small and after performing second order perturbation theory [18] we find that

$$\epsilon_0(\lambda) \approx \lambda P_{mean} - \lambda^2 \sum_{n \neq 0} \frac{V_{n0}^2}{\epsilon_n} \quad (25)$$

where V_{n0} is the expectation of $V(\mathbf{r})$ over the eigenfunction $\phi_{n0}(\mathbf{r})$ of the Laplacian and ϵ_{n0} the corresponding eigenvalue in the square domain. It then turns out that $I(x)$ takes the form

$$I_e(x) \approx \frac{(x - P_{mean})^2}{4\sigma^2} \quad (26)$$

with σ^2 is the term multiplying λ^2 above, recovering [19].

In Fig. 1(a) we plot the numerically generated rate function of the energy $I_e(x)$ by calculating the minimum eigenvalue $\epsilon_0(\lambda)$ of (11) with $V(r) = \lambda \gamma r^\beta$ in a square of unit length for various values of λ . Then for any given value of x , we use this function to find the minimum of $\lambda x - \epsilon_0(\lambda)$. This minimum value is plotted in the figure. The same is done for a unit length line by finding the minimum eigenvalue of the same operator. We see that $I_e(x)$ vanishes when $x = P_{mean} =$

$\mathbb{E}_{\mathbf{r}}[P(\mathbf{r})]$. The rate function plotted provides information about the distribution of the energy E_T as discussed in (10). Indeed for $E_T > P_{mean}T$ the probability distribution of E_T is (to logarithmic accuracy) given by

$$\mathbb{P}(E_T > xT) \sim e^{-I_e(x)T}. \quad (27)$$

V. THROUGHPUT STATISTICS

Next, the statistics of uplink user throughput are given. As discussed earlier, the appropriate functional here is the integrated rate given by $B_T = \int_0^T C(\mathbf{r}(t))dt$. As in the previous section, this is used to obtain the rate function $I_b(x)$, which provides the leading (exponential) term in the distribution of the total transmitted bits B_T for large T .

In Fig. 1(b) we plot the numerically generated rate function of the throughput $I_b(B_T/T)$ evaluated similarly as in the previous section for the energy. The larger difference between one- and two-dimensional values of the rate for larger values of the throughput are due to the lower density close to the center in two-versus one-dimensional geometries. As also discussed above, the rate function $I_b(x)$ provides to logarithmic accuracy the probability that $B_T < xT$, thus providing a metric for the “outage” probability of the total throughput. Indeed, when $B_T < C_{mean}T$ with $C_{mean} = \mathbb{E}_{\mathbf{r}}[C(\mathbf{r})]$ being the value of $x = B_T/T$, $I_b(x) = 0$ and the probability distribution of B_T is, to logarithmic accuracy equal to

$$\mathbb{P}(B_T > xT) \sim e^{-I_b(x)T}. \quad (28)$$

VI. DISCUSSION AND OUTLOOK

In summary, we have introduced a methodology to obtain the distribution tails of performance metrics, such as the total throughput and the consumed energy over time by exploiting the statistics of mobility. This can help improve network design and dimensioning, by providing analytic results for low probability events. As specific examples, we calculated the long time distribution of the consumed uplink power and the corresponding total throughput of a single user in a massive MIMO cell under the assumption of Brownian motions.

We can generalize the above discussion for Levy random Markov processes that have infinite variance of each step, corresponding to long tailed distributions [10], [11]. The only difference is the introduction of an appropriate infinitesimal generator of the process, which is a symmetric stable law of index $\alpha < 2$ [20]. In this case \mathbf{M}_0 does not have a local representation (as a derivative), but is still well defined [17], [20], [21]. It is worth mentioning briefly how the above results are expected to change, focusing for brevity only on the energy case. Since \mathbf{M}_0 will have scale dimensions of $R^{-\alpha}$, we can rescale the equations to find that for large positive λ the minimum eigenvalue will be $\epsilon_0(\lambda) \sim \lambda^{\alpha/(\alpha+\beta)}$. Conversely, for large negative λ the minimum eigenvalue will be $\epsilon_0(\lambda) - \lambda r_{max}^{\beta} \sim \lambda^{\alpha/(1+\alpha)}$. Putting all this together leads to a rate function

$$I_{e,low}(x, \alpha) \sim x^{-\alpha/\beta} \quad (29)$$

$$I_{e,high}(x, \alpha) \sim (1 - xr_{max}^{-\beta})^{-\alpha}. \quad (30)$$

The above results can also be generalized to larger systems with many BSs. In such situations, a user switches between BSs when crossing the cell boundary in which case the energy consumption in the uplink continues to increase, or in the downlink the power associated to that user is switched off. Mathematically, this has the effect of having a periodic power function, when the cells are assumed to appear in an ordered fashion. Another obvious generalization has to do with taking into account orthogonal (such as OFDMA) channels and treating the total downlink sum-throughput and/or power consumption. In this system, we end up with an operator \mathbf{M} describing multiple Brownian motions.

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